Physical World

Chapter 1

1 SCIENCE

Systematic attempt to understand natural phenomena in as much detail and depth as possible, and use the knowledge so gained to predict, modify and control phenomena

2 SCIENTIFIC METHOD

The scientific method involves several interconnected steps:

- Systematic observations
- Controlled experiments,
- Qualitative and quantitative reasoning
- Mathematical modelling, prediction and verification or falsification of theories

3 HYPOTHESIS AND AXIOMS

- A hypothesis is a supposition without assuming that it is true.
- An axiom is a self-evident truth while a model is a theory proposed to explain observed phenomena.

4 PHYSICS

Study of the basic laws of nature and manifestation in different natural phenomenon

5 PRINCIPAL THRUSTS

Unification

To explain diverse physical phenomenon in terms of few concepts and laws.

Reduction

To derive the properties of a larger and a more complex problem or system into simpler constituents

6 PRINCIPAL DOMAINS

Macroscopic domain

Phenomenon at laboratory, terrestrial and celestial scale Mainly dealt by classical physics including mechanics, electrodynamics, optics and thermodynamics

Microscopic domain

Constitution and structure of matter at the minute scales of atoms and nuclei. Mainly dealt by quantum physics

8 NATURE OF PHYSICAL LAWS

- **Conservation of energy :** In an isolated system, total energy remains conserved.
- **Conservation of linear momentum :** In an isolated system, total linear momentum remains conserved.
- **Conservation of angular momentum :** In an isolated system, total angular momentum remains conserved.
- Symmetry of nature with respect to translation in time is equivalent to the law of conservation of energy
- Symmetry of the laws of nature with respect to translation in space gives rise to conservation of linear momentum
- Isotropy of space (no intrinsically preferred direction in space) underlies the law of conservation of angular momentum

7 FUNDAMENTAL FORCES IN NATURE

Strong nuclear force

- Acts between nucleons
- Short range (Nuclear size $\approx 10^{-15}$ m)
- Relative strength = 1
- Mediating particles are mesons

Electromagnetic force

- Force due to virtue of charges
- Both attractive and repulsive
- Range is infinite
- Relative strength = 10^{-2}
- Mediating particles are photons

Weak Nuclear Force

- Between some elementary particles particularly electron and neutrino
- Very short range ($\approx 10^{-16}$ m)
- Relative strength = 10^{-13}
- Mediating particles are bosons

Gravitational Force

- Force of attraction by virtue of mass
- Always attractive in nature
- It is weakest fundamental force
- Range is infinite
- Relative strength = 10^{-36}
- Mediating particle are graviton

Units and Measurements

Chapter 2

1 UNITS

Measurement of any physical quantity involves comparison with certain basic arbitrarily chosen internationally accepted reference called units.

Classification

- **Fundamental units**: Independent of each other
- **Derived units**: Expressed as combination of fundamental units

- A complete set of these units, both the base units and derived units is known as system of units.
- Old system of units: CGS, FPS and MKS system.
- In **CGS** fundamental units are centimeter, gram and second.
- In **FPS** fundamental units are foot, pound and second.
- In **MKS** fundamental units are meter, kilogram and second.

3 MEASUREMENT OF LENGTH

- Large distance is measured by parallax method.
- Parallax angle = $\frac{\text{Basis}}{\text{Distance}}$
- $1° = 1.745 \times 10^{-2}$ rad
- $1'' = 4.85 \times 10^{-6}$ rad
- Measurement of very small distance like size of molecule uses, Optical microscope, Electronic microscope and Tunneling microscope
- 1 AU = 1.496×10^{11} m
- 1 ly = 9.46×10^{15} m
- 1 parsec = 3.08×10^{16} m
- Size of proton 10^{-15} m
- Radius of Earth 10^{7} m
- Distance to boundary of observable universe 10^{26} m

4 MEASUREMENT OF MASS

- SI unit is kilogram (kg)
- Unified atomic mass unit (u). It is used to measure mass of atoms and molecules
- 1 u = 1/12 × mass of one C-12 atom.
- 1 u = 1.66×10^{-27} kg
- Electron mass 10^{-30} kg
- Earth mass 10^{25} kg
- Observable universe 10^{55} kg

5 MEASUREMENT OF TIME

- Atomic standard of time: This is based on caesium clock, uncertainty gained overtime by caesium atomic clock is less than 1 part in 10^{13} (loss of 3 μs in one year)
- Time span of most unstable particle 10^{-24} s
- Travel time for light from nearest star 10^{8} s
- Age of universe 10^{17} s

6 ACCURACY and PRECISION

- Every measurement by any measuring instrument contains some uncertainty called error.
- Accuracy of a measurement is a measure of how close is the measured value to true value.
- Precision tells us to what resolution the quantity is measured.
- It is not necessary that more precise value is more accurate too.

2 SI SYSTEM OF UNITS (INTERNATIONAL SYSTEM OF UNITS)

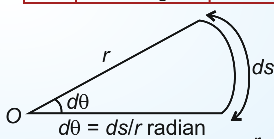
- Presently accepted internationally for measurement is SI system of units, revised in 2018. Certain rules to follow with standard symbols
- It is decimal system thus, conversion within system is easy and convenient
- It has 7 base unit and 2 supplementary units

Base Units

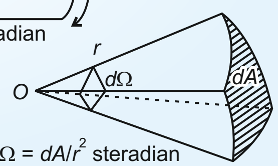
S.N.	Quantity	Unit	Symbol
1.	Length	meter	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Electric current	ampere	A
5.	Thermodynamic temperature	kelvin	K
6.	Amount of substance	mole	mol
7.	Luminous intensity	candela	cd

Supplementary Units

S.N.	Quantity	Unit	Symbol
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr



$d\theta = ds/r$ radian



$d\Omega = dA/r^2$ steradian

7 ERRORS IN MEASUREMENT

Errors

- Systematic
  - Instrumental
  - Experimental
  - Personal
- Random

- Every measurement is approximate due to errors.
- Random errors occurs irregularly.
- Least count error is smallest value that can be measured by instrument (occurs within both systematic and random errors).
- Absolute error = $\frac{\Sigma(|a_i - a_{mean}|)}{n}$
- Relative error = $\frac{\Delta a_{mean}}{a_{mean}}$
- Percentage error = $\frac{\Delta a_{mean}}{a_{mean}} \times 100$

Combination of errors

- Sum and difference: $\Delta Z = \Delta A + \Delta B$
- Product or Quotient: $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

- If $X = \frac{A^a B^b}{C^c}$ then $\% \frac{\Delta x}{x} = a\left(\% \frac{\Delta A}{A}\right) + b\left(\% \frac{\Delta B}{B}\right) + c\left(\% \frac{\Delta C}{C}\right)$

8 SIGNIFICANT FIGURES

- Reliable digits plus first uncertain digit are known as significant digit.
- A choice of change of different units does not change number of significant digits.
- All non-zero digits are significant.
- All zero between two non-zero digits are significant.
- The terminal zeros in a number without a decimal point are not significant.
- The trailing zeros in a number with decimal point are significant.

Rules of Arithmatic Operations with Significant Figures

- Addition/Substraction: Final result contains as many decimal places as in number with least decimal places.
  e.g. 3.307 + 0.52 = 3.83
- Multiplication/Division: Result contains as many significant figures as in number with least number of significant figures.
  e.g. 4.11/1.2 = 3.4

Rounding off

- Preceding digit is raised by 1 if insignificant digit to be dropped is more than 5 and left unchanged of latter is less than 5.
- If insignificant digit is 5 then preceding digit is left unchanged if its even and uncreased by 1 if it is odd.

10 DIMENSIONAL FORMULAE AND SI UNITS OF VARIOUS PHYSICAL QUANTITIES

S. No.	Physical Quantity	Relation with other quantities	Dimensional Formula	SI Unit
1.	Gravitational constant 'G'	$\frac{\text{Force} \times (\text{distance})^2}{\text{Mass} \times \text{mass}}$	$\frac{[MLT^{-2}][L^2]}{M \times M} = [M^{-1}L^3T^{-2}]$	N m^2 kg^{-2}
2.	Stress	$\frac{\text{Force}}{\text{Area}}$	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$	N m^{-2}
3.	Coefficient of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$\frac{ML^{-1}T^{-2}}{1} = [ML^{-1}T^{-2}]$	N m^{-2}
4.	Surface tension	$\frac{\text{Force}}{\text{Length}}$	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$	N m^{-1}
5.	Coefficient of viscosity	$\frac{\text{Force} \times \text{distance}}{\text{Area} \times \text{velocity}}$	$\frac{MLT^{-2} \times L}{L^2 \times LT^{-1}} = [ML^{-1}T^{-1}]$	N m^{-2} or Pa s or decapoise
6.	Planck's constant '*h*'	$\frac{E}{\nu} = \frac{\text{Energy}}{\text{Frequency}}$	$\frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$	J s
7.	Velocity gradient	$\frac{\text{Velocity}}{\text{Distance}}$	$\frac{LT^{-1}}{L} = T^{-1} = [M^0L^0T^{-1}]$	s^{-1}
8.	Pressure gradient	$\frac{\text{Pressure}}{\text{Distance}}$	$\frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$	Pa m^{-1}

9 DIMENSIONAL ANALYSIS

Dimensions

- Nature of physical quantity is determined by its dimension.
- The dimensions of physical quantity are powers to which base quantities are raised to represent it.
- The dimension of time in speed is –1.

Dimensional equation

- The expression which shows how and which of the base quantities represent the dimension of physical quantity is called dimensional formula.
- An equation is obtained by equating physical quantity with its dimensional formula.
- For example $[A] = [M^0L^2T^0]$

Homogeneity principle

Physical quantities represented by symbols on both sides of a mathematical equation must have same dimensions.

Applications

Checking dimensional consistency of equations

- It is based on homogeneity law. An equation is dimensionally correct if dimension of fundamental quantities of each term on left side of equation is equal to that on right hand side.

Deducing relations among physical quantities.

- We should know the dependence of physical quantity on other upto three physical quantities and product type of dependence

Limitations of Dimensional Analysis

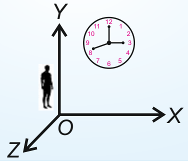
- Dimensional analysis is useful in deducing relations among inter dependent physical quantities but dimensional constant can not be determined.
- It can test dimensional validity but not exact relationship between physical quantities having same dimensions.
- It does not distinguish between the physical quantities having same dimensions.

Motion in a Straight Line

Chapter 3

1 FRAME OF REFERENCE

- A rectangular coordinate system consisting of three mutually perpendicular axes, along with a clock. The point of intersection of these three axes is called origin (O)
- If a body changes its position as time passes w.r.t. frame of reference, it is said to be in motion.
- Motion of objects along a straight line is called rectilinear motion.



2 DISTANCE AND DISPLACEMENT

- **Distance:** Actual path length in motion. During motion it is non-zero
- **Displacement:** The shortest path between initial and final position. Equal to change in position. May or may not be equal to path length travelled. It can be positive, negative or zero.

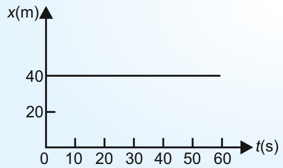


Fig: Stationary object

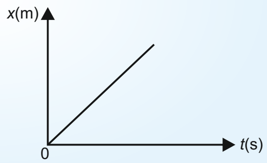


Fig: Object in uniform motion

3 SPEED

- The rate of distance covered with time is called speed,

$$v = \frac{\text{distance}}{\text{total time}} = \frac{\ell}{t}$$

Average Speed

$$v_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{\text{total path length}}{\text{total time interval}}$$

Instantaneous speed

$$v = \lim_{\Delta t \to 0} \frac{\Delta \ell}{\Delta t} = \frac{d\ell}{dt}$$

4 VELOCITY

- The rate of change of position, It tells how fast position is changing with time and in what direction.

Average velocity

$$\vec{v}_{av} = \frac{\Delta \vec{x}}{\Delta t}$$

- SI units are m s^{-1}

Instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

- Slope of position time graph

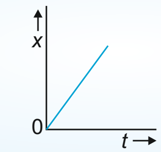


Fig: Moving with positive velocity

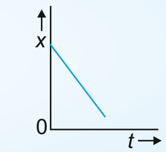


Fig: Moving with negative velocity

5 ACCELERATION

The time rate of change of velocity

Average Acceleration

$$\vec{a}_{av} = \frac{(v_2 - v_1)}{(t_2 - t_1)} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Uniform Acceleration

Equal change in velocity in equal intervals of time

Non-Uniform Acceleration

Acceleration changes with time

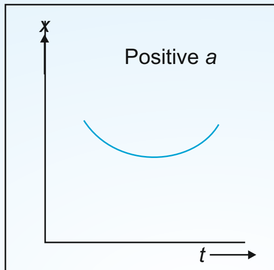


Fig: Positive acceleration

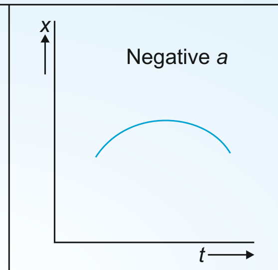


Fig: Negative acceleration

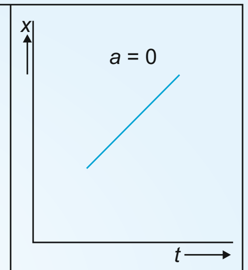


Fig: Zero acceleration

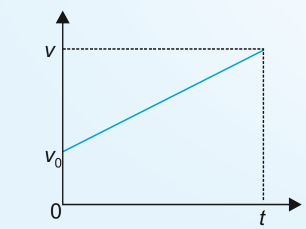


Fig: Motion in positive direction with positive acceleration

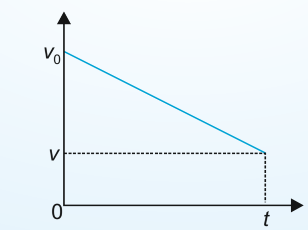


Fig: Motion in positive direction with negative acceleration

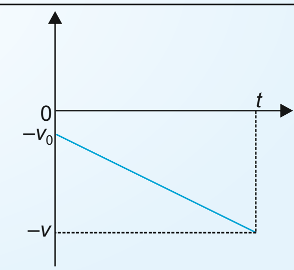
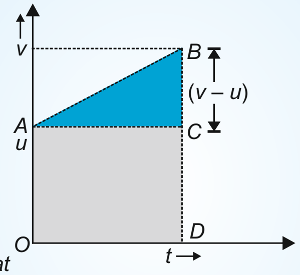


Fig: Motion in negative direction with negative acceleration

6 Kinematic Equations

- A mathematical treatment to describe the motion of a body in one-dimension.

For uniformly accelerated motion



- $v = u + at$
- $s = ut + \frac{1}{2}at^2 = \left(\frac{u+v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s_n = u + \frac{a}{2}(2n-1)$
- $\bar{v} = \frac{u+v}{2}$

7 FOR MOTION WITH VARIABLE ACCELERATION

- $\frac{dv}{dt} = a \Rightarrow v - u = \int a dt$
- $\frac{dx}{dt} = v \Rightarrow \Delta x = \int v dt$ (Area under $v-t$ curve)
- $\frac{vdv}{dx} = a$
- $\frac{d^2x}{dt^2} = a$

8 FOR MOTION UNDER GRAVITY

- A mathematical treatment to describe the motion of a body in one-dimension under free fall

Vertically downward motion

When object is released from $y = 0$

- $v = -gt$
- $y = -\frac{1}{2}gt^2$
- $v^2 = -2gy$

Vertically upward motion

$u \neq 0$, acceleration $a = -g$

- $v = u - gt$
- $S = ut - \frac{1}{2}gt^2$
- $v^2 = u^2 - 2gh$
- Distance travelled during equal intervals of time by a body falling freely from rest is in ratio 1 : 3 : 5 : 7 : 9 : 11 .. (Galileo's law)

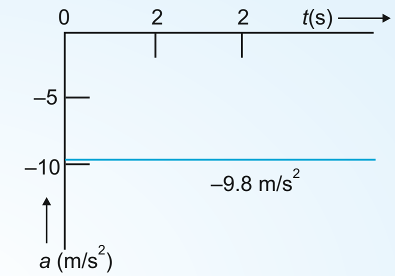


Fig: Variation of acceleration with time

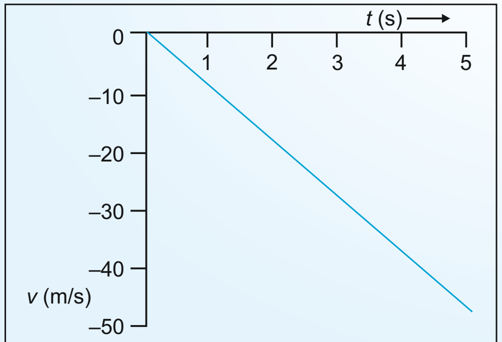


Fig: Variation of velocity with time

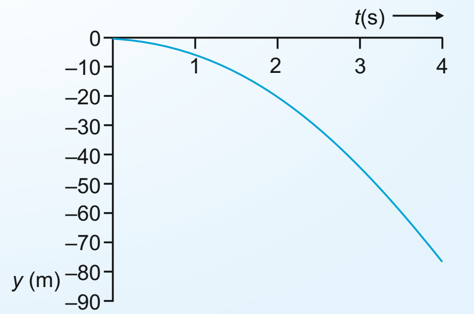


Fig: Variation of distance with time

9 Relative Velocity

- The velocity with which an object moves with respect to another object is called relative velocity.

$v_{AB} = (v_A - v_B)$

$v_{AB} = (v_A + (-v_B))$

$v_{BA} = (v_B - v_A)$

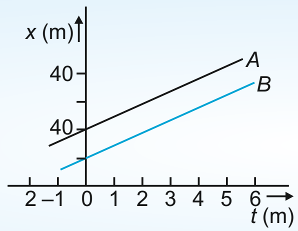


Fig: Position-time graphs of two objects with equal velocities

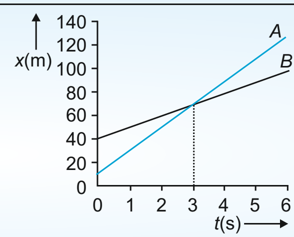


Fig: Position-time graphs of two objects with unequal velocities, showing the time of meeting

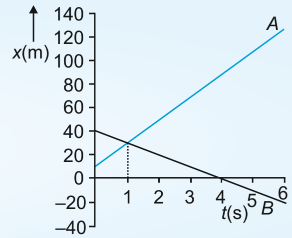


Fig: Position-time graphs of two objects with velocities in opposite directions, showing the time of meeting

Motion in a Plane

Chapter 4

1 SCALARS AND VECTORS

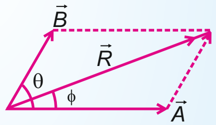
- **Scalar quantity:** It has only magnitude with proper unit. All base quantities are scalar. The rules combining scalars are rules of ordinary algebra.
- **Vector quantity:** It has both magnitude and direction and obeys the triangle law or parallelogram law of vector addition.
- **Equality of vector:** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal, if and only if, they have same magnitude and direction.
- **Multiplication of vector by real numbers:** If a vector $\vec{A}$ is multiplied by real number λ, then $A' = \lambda|\vec{A}|$

  if $\lambda > 0$, magnitude will change and direction remains same

  if $\lambda < 0$, magnitude changes λ times and direction gets reverse.
- **Parallelogram law of vector addition:** For two co-initial vectors represented by two adjacent sides of a parallelogram, the diagonal of a parallelogram passing through same point will be resultant.

  $|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

  $\tan\phi = \dfrac{B\sin\theta}{A + B\cos\theta}$

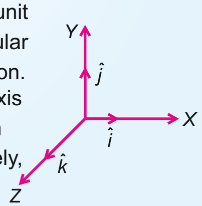


- **Subtraction of vector:** It can be defined as addition of a vector and negative of other vector.

  $\vec{S} = \vec{A} - \vec{B}$

  $\vec{S} = \vec{A} + (-\vec{B}) \Rightarrow |\vec{S}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

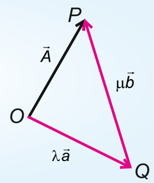
Unit Vectors: It is a vector of unit magnitude and points in a particular direction. It has no unit and dimension. Unit vectors along the x, y and z axis of a rectangular coordinate system represented by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively, called basic unit vectors.



2 RESOLUTION OF VECTORS

$\vec{A} = \vec{OP} = \vec{OQ} + \vec{QP}$

$\vec{A} = \lambda\vec{a} + \mu\vec{b}$



3 RECTANGULAR COMPONENTS

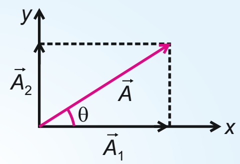
- $\vec{A} = \vec{A}_1 + \vec{A}_2$

  $\vec{A} = A_x\hat{i} + A_y\hat{j}$

  $\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$

  $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

  $\tan\theta = \dfrac{A_y}{A_x}, \ \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$

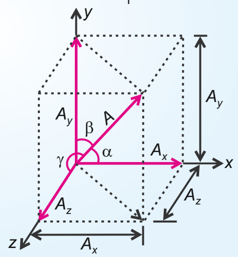


- Resolution in three. rectangular components

  $A_x = A\cos\alpha, \ A_y = A\sin\alpha$

  $A_z = A\cos\gamma$

  $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



4 MOTION IN A PLANE

$\vec{r} = x\hat{i} + y\hat{j}$

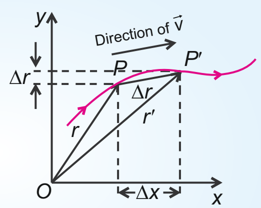
$\vec{r}' = x'\hat{i} + y'\hat{j}$

$\vec{\Delta r} = \vec{r}' - \vec{r}$

$\vec{\Delta r} = (x' - x)\hat{i} + (y' - y)\hat{j}$

$\vec{v}_{av} = \dfrac{\vec{\Delta r}}{\Delta t} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j}$

Instantaneous velocity, $\vec{v} = \dfrac{d\vec{r}}{dt}$



- The direction of velocity at any point on path is tangent to path and in direction of motion.

5 MOTION IN A PLANE WITH CONSTANT ACCELERATION

- $\vec{v} = \vec{v}_o + \vec{a}t$

  $v_x = v_{ox} + a_x t$

  $v_y = v_{oy} + a_y t$
- $\vec{r} = \vec{r}_o + \vec{v}_o t + \dfrac{1}{2}\vec{a}t^2, \quad x = x_o + v_{ox}t + \dfrac{1}{2}a_x t^2$

  $y = y_o + v_{oy}t + \dfrac{1}{2}a_y t^2$

6 RELATIVE VELOCITY IN TWO DIMENSIONS

The velocity of object A relative to B

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

where $\vec{V}_A$ and $\vec{V}_B$ are velocities in the same frame.

Similarly, $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$\vec{V}_{AB} = -\vec{V}_{BA}$ and $|\vec{V}_{AB}| = |\vec{V}_{BA}|$

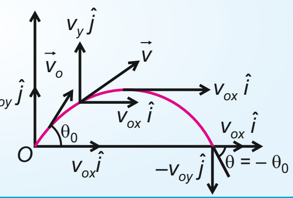
7 PROJECTILE MOTION

Equation of trajectory $y = x\tan\theta_0 - \dfrac{1}{2}\dfrac{gx^2}{v_o^2\cos^2\theta_0}$

This is equation of parabola.

- Time of flight $T_f = \dfrac{2v_o\sin\theta_0}{g}$
- Maximum height $h_m = \dfrac{(v_o\sin\theta_0)^2}{2g}$
- Horizontal range $R = \dfrac{v_o^2\sin 2\theta_0}{g}$

  for R_{max}, $\theta = 45°$, $R_{max} = \dfrac{v_o^2}{g}$

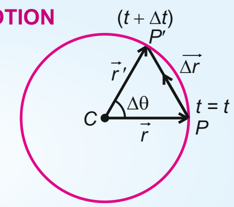


8 UNIFORM CIRCULAR MOTION

In uniform circular motion particle moves with constant speed.

- Angular displacement $\Delta\theta = \dfrac{\text{Arc }(PP')}{r}$
- Angular velocity $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{2\pi}{T} = 2\pi\nu$
- Linear speed $v = r\omega$
- Centripetal acceleration-Due to change in direction of velocity and is always directed towards centre.

  $a = \dfrac{v^2}{r} = r\omega^2 = 4\pi^2\nu^2 r = v\omega$



Laws of Motion

5
Chapter

1 NEWTON'S 1ST LAW

A body continues its state of rest or of motion until unless an external force is acted on it

Inertia of rest
The property of body due to which it cannot change its state of rest by itself.

Inertia of motion
The property of body due to which it cannot change its state of motion by itself.

Inertia of direction
The property due to which a body cannot change its direction of motion by itself.

2 NEWTON'S 2ND LAW

The rate of change of Linear momentum of a body is directly proportional to the external force applied on the body and takes place in the direction in which force acts

$$F = \frac{dp}{dt} = ma$$

- The same force for the same time causes same change in momentum for different bodies.

Impulse
A large force acts for very short duration of time produces a finite change in momentum. Product of force and time duration for which it acts is impulse.

Impulse $= F \times \Delta t = \Delta p$

Equilibrium of a particle

$\Sigma \vec{F} = 0 \Rightarrow \Sigma \vec{F}_x = 0, \Sigma \vec{F}_y = 0$ and $\Sigma \vec{F}_z = 0$

Conservation of Liner Momentum
Total momentum of an isolated system of intracting particles is conserved if there is no external force acting on it.

$$\vec{p}_{initial} = \vec{p}_{final}$$

4 NON-INTERTIAL FRAME OF REFERENCE

Pseudo Force $\vec{F}_{pseudo} = -M\vec{a}_{frame}$

$$\vec{F}_{ext} + \vec{F}_{pseudo} = M\vec{a}$$

3 NEWTON'S 3RD LAW

To every action there is always an equal and opposite reaction

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

- Forces always occur in pairs. Force on body A by B is equal and opposite to force on body B by A.

Some examples of Newton's 3rd Law

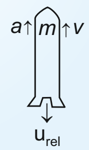
- Recoiling of Gun
- Rowing of boat
- When a man jumps from a boat, the boat moves backward
- It is difficult to walk on sand or ice.

Rocket Propulsion

- $a = \frac{u_{rel}}{m}\frac{dm}{dt} - g$

Thrust

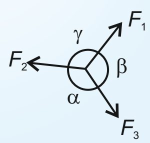
- $F = -u_{rel}\frac{dm}{dt}$



7 PROBLEM SOLVING TECHNIQUES IN MECHANICS

- Identify the unknown forces and accelerations
- Draw FBD of bodies in system
- Resolve forces into components
- Apply $\Sigma \vec{F} = 0$ in the direction of equilibrium
- Apply $\Sigma \vec{F} = M\vec{a}$ in the direction of accelerated motion
- Write constraint relations if exists.
- Solve the equations $\Sigma \vec{F} = 0$ and $\Sigma \vec{F} = M\vec{a}$
- For equilibrium of concurrent forces use sine rule

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



5 COMMON FORCES IN MECHANICS

Tension Force

- Restoring force in string is called tension.
- It is due to electromagnetic force
- Always acts away from the body
- It is a contact force.

Weight

- It is equal to the gravitational pull *i.e.* $W = Mg$
- It is non-contact force.

Normal Reaction
It is always perpendicular to the surface in contact.

- It is a contact force.

Spring Force

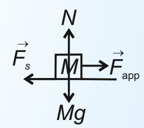
- $\vec{F} = -K\vec{x}$
- It is due to electromagnetic force
- It is a contact force.

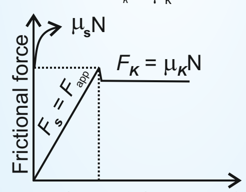
Friction
It is the resistance offered to the relative motion between two bodies in contact

- It is parallel to surface of body in contact.

Type of Friction

- Static friction: $F_s = F_{applied}$
- Limiting friction $F_{lim} = \mu_s N$
- Kinetic friction $F_k = \mu_k N$





$(F_s = F_{app})$ applied force

- Acceleration of body sliding down a rough inclined plane
  $a = g(\sin\theta - \mu\cos\theta)$
- Angle of friction: $\theta = \tan^{-1}(\mu_s)$
- Angle of repose: $\alpha = \tan^{-1}(\mu_s)$

6 CIRCULAR MOTION

A body moving in a circular path is called circular motion.

$F_c = mv^2/R$ is called centripetal force.

Uniform circular motion

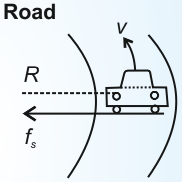
- $a = a_c = \frac{v^2}{R} = R\omega^2$
- $a = a_c = v.\omega$

Non-uniform circular motion

- $\vec{a} = \vec{a}_T + \vec{a}_c$
- $a = \sqrt{a_T^2 + a_c^2}$

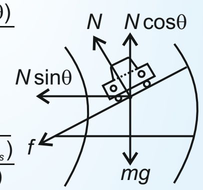
Motion of car on level Road

- $v_{max} = \sqrt{\mu_s Rg}$
- $\mu_{min} = \frac{v^2}{Rg}$
- $R_{min} = \frac{v^2}{\mu g}$



Motion of car on Banked Road

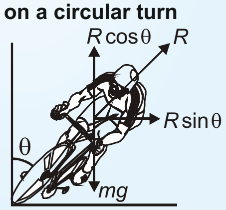
- $v_{max} = \sqrt{\frac{Rg(\mu_s + \tan\theta)}{(1 - \mu_s\tan\theta)}}$
- $v_{optimum} = \sqrt{Rg\tan\theta}$
- $v_{min} = \sqrt{\frac{Rg(\tan\theta - \mu_s)}{(1 + \mu_s\tan\theta)}}$



Bending of cyclist on a circular turn

- Angle of Bending

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$



- Numerically: $\alpha = \theta$
- Kinetic friction is usually less than maximum value of static friction.

Work, Energy and Power

6

Chapter

1 SCALAR PRODUCT

- Also called dot product
- $\vec{A}\cdot\vec{B} = AB\cos(\theta)$
- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
- $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- $(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = A_xB_x + A_yB_y = A_zB_z$

2 WORK

- Scalar product of force and displacement is work.
- Work done by a force can be positive, negative or zero.
- Work done by gravity in horizontal displacement of object is zero.
- Work done by tension in pendulum bob is zero.
- Work done by spring elastic force during stretching or compressing is negative.
- Work can be done by a constant or variable force.

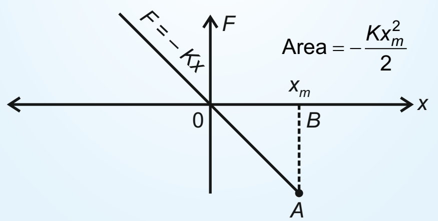
A. Constant force $W = \vec{F}\cdot\Delta\vec{r}$

B. Variable force $W = \int_{r_1}^{r_2} \vec{F}(r)\cdot d\vec{r}$

C. When force-displacement graph is given, Area under force-displacement curve gives work done by the force.

D. Work done in stretching a spring by distance Δl is

$W = \frac{1}{2}K(\Delta l)^2$



(Potential energy) $U = -W_S = \frac{1}{2}kx_m^2$

3 ENERGY

- Capability of an object to perform work is its energy.

Some common units of energy

Kilowatt hour	3.6×10^{6} J
erg	10^{-7} J
Electron volt	1.6×10^{-19} J
Calorie	4.186 J

Mechanical energy is of two forms

Kinetic Energy

- Energy a body possesses by virtue of its motion.
- $KE = \frac{1}{2}mv^2$
- Its unit is joule
- Work is related to KE of body by theorem called work-energy theorem.

$$\Delta K = K_f - K_i = W = \int_{x_1}^{x_2} F(x)\,dx$$

- This theorem is in scalar form.
- Shape of graph between KE of a body and its speed is parabola.
- $(F_c + F_{nc})\,\Delta x = \Delta K$, when both forces are present

  F_c = Conservative forces

  F_{nc} = Non-conservative forces
- Kinetic energy of a body of fixed mass is directly proportional to square of its momentum.

$$K = \frac{P^2}{2m}$$

- Kinetic energy of fast moving air is used to generate electricity in wind mill.
- If two objects have same momentum, then the lighter has more kinetic energy and vice versa.
- Kinetic energy of fast flowing stream has been used to grind corn and now to generate hydro-electricity.

Potential Energy

- It is form of stored energy, by virtue of position or configuration of body.
- Notion of potential energy is applicable to class of conservative forces. Work done against such forces gets stored up as potential energy. When constraints are removed, this energy may appear as kinetic energy.
- Change in potential energy for a conservative force; ΔU is equal to negative of work done by the force
- $\Delta U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r}$
- A force is conservative if it is derived from a scalar quantity $U(x)$ by relation $F_x = -\frac{dU}{dx}$
- Work done by a conservative force depends only on initial and final points. Zero of potential energy is arbitrary.
- Work done by gravity depends on initial and final position only $U_h = mgh$ (Gravitational potential energy at height h)
- Potential energy of a stretched spring

$$U = \frac{Kx_m^2}{2}$$

K is spring constant. Spring is said to be stiff if K is high, x_m = extension of spring.

4 LAW OF CONSERVATION MECHANICAL ENERGY

- If conservative and non-conservative forces acts on a body then

$(F_c + F_{nc})\Delta x = \Delta K$

Now, $F_c \Delta x = -\Delta U$

$\Delta(K + U) = F_{nc}\Delta x$

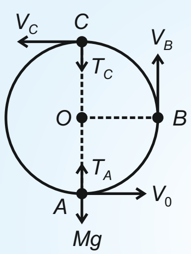
$\boxed{\Delta E = F_{nc}\Delta x}$

E = Total Mechanical Energy (Consequence of work energy theorem). If $F_{nc} = 0$ then $\boxed{\Delta E = 0}$

- Mechanical energy of a system is conserved if the forces doing work on it are conservative.

Conservative Forces	Non-conservative Forces
The work done by or against the force in moving a body depends only on initial and final position of the body and not on path followed in between.	The work done by or against the force in moving a body from one position to another depends on the path followed between the initial and final positions.

5 VERTICAL CIRCULAR MOTION



$$T_A = \frac{MV_0^2}{L} + Mg \text{ and } E_A = \frac{1}{2}MV_0^2$$

$$E_C = \frac{1}{2}MV_C^2 + 2MgL = \text{constant}$$

$$Mg = \frac{MV_C^2}{L}$$ when string slackens (just completes loop)

$$E_A = E_C = \frac{5}{2}MgL = \frac{MV_0^2}{2}$$

$$\therefore \; V_0 = \sqrt{5gL}$$

Minimum speed at different locations to complete loop $V_C = \sqrt{gL}$

$$V_B = \sqrt{3gL}$$

$$K_A : K_B : K_C = 5 : 3 : 1$$

7 COLLISION

- Exchange of momentum between objects is consequence of collision, due to material impulsive forces. The laws of momentum and energy conservation are used in collision.
- Collision are classified as elastic and inelastic collision depending on nature of colliding bodies.
- In all collisions, total linear momentum of the systemis conserved. Initial momentum of system is equal to final momentum of the system.

COLLISION → PERFECT INELASTIC COLLISION / INELASTIC / ELASTIC COLLISION

PERFECT INELASTIC COLLISION

A collision in which two colliding particles move together (in one dimensions) after the collision is complete inelastic collision. Kinetic energy is always lost in such collisions.

For m_1 moving at u_1 and m_2 at rest.

$$v_f = \frac{m_1u_1}{m_1 + m_2}$$

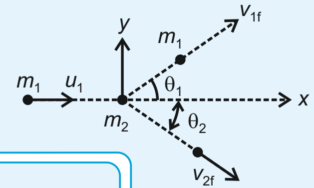
Loss in KE on collision

$$\boxed{\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1)^2}$$

INELASTIC

An intermediate collision case where the deformation is partly restored and some of initial kinetic energy is lost. Momentum of two colliding bodies before and after will remain conserved.

- In two dimension, inelastic collision if target at rest, two object don't move at right angles to each other (glancing collision) even when identical.



ELASTIC COLLISION

Both linear momentum and kinetic energy of system of colliding particles will remain conserved.

- In one dimensional collision with target initially at rest

$$\boxed{v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1}$$

$$\boxed{v_{2f} = \frac{2m_1u_1}{m_1 + m_2}}$$

- If two masses are equal and target is at rest

$$\boxed{v_{1f} = 0, \quad v_{2f} = u_1}$$

First one comes to rest and pushes off the second with its initial speed. Thus, velocities are exchanged

- If $m_2 >> m_1$

$$\boxed{v_{1f} = -u_1 \text{ and } v_{2f} \simeq 0}$$

Heavier mass is undisturbed while lighter mass reverses its velocity.

- Fraction of kinetic energy lost by targetting body when target at rest

$$f_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

and fraction of KE gained by target being at rest initially

$$f_2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

- When two equal masses undergo two dimensional elastic collision with one of them at rest, after collision they will move at right angles to each other.

6 POWER

- Rate at which work is done is power.
- $P = \frac{dW}{dt} = F \cdot \frac{dr}{dt} = \vec{F} \cdot \vec{v}$
- Rate at which energy is transferred is power.
- Average power is ratio of total work to total time taken.
- $P_{av} = \frac{W}{t}$
- SI unit of power is watt.
- Another unit of power is horse power.

[1 hp = 746 W]

- A machine which performs same amount of work over a shorter period of time has more power.

System of Particles and Rotational Motion

Chapter 7

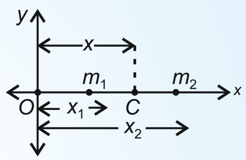
1 RIGID BODY

- Ideally a rigid body is a body with a perfectly definite and unchanging shape. The distances between all pairs of particles of such a body do not change.
- In pure translational motion at any instant of time all particles of the body have same velocity.
- The motion of rigid body which is pivoted or fixed is rotation. Every particle of the body moves in a circle.
- The motion of rigid body which is not pivoted or fixed in some way is either a pure translation or is combination of translation and rotation

2 CENTRE OF MASS

- COM is an imaginary point where mass of an extended body is assumed to be concentrated
- This concept is used to study indepedently translatory and rotatory motion under effect of external forces.
- The laws of motion which are applied to particles can be applied to large sized bodies by converting body into a particle at location of COM.
- **Centre of mass for two particle system**

$$\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$



- For x and y plane

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \text{ and, } Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

- For a system of n particles distributed in space,

$$X_{cm} = \frac{\Sigma m_i x_i}{M}, Y_{cm} = \frac{\Sigma m_i y_i}{M}, Z_{cm} = \frac{\Sigma m_i z_i}{M}$$

- **COM For Continous Mass**

If the body has continous distribution of mass (RING, DISC, ROD)

$$\vec{R} = \frac{1}{M}\int \vec{r}dm \quad M = \text{total mass of body}$$

The co-ordinates of COM of body,

$$X_{cm} = \frac{1}{M}\int xdm, Y_{cm} = \frac{1}{M}\int ydm, Z_{cm} = \frac{1}{M}\int zdm$$

- If we choose centre of mass at origin $\int \vec{r}dm = 0, \int xdm = \int ydm = \int zdm = 0$
- For homogeneous bodies of regular shape, centre of mass lies at geometric centre.

3 MOTION OF COM

$$M\vec{R} = \Sigma m_i\vec{r}_i$$

$$\therefore \; M\vec{V} = \Sigma m_i\vec{v}_i$$

- Velocity of COM of system

$$\therefore \; \vec{V} = \frac{\Sigma m_i\vec{v}_i}{M}$$

- Acceleration Of Com of System

$$\vec{A} \text{ or } \vec{a}_{cm} = \frac{\Sigma m_i\vec{a}_i}{M}$$

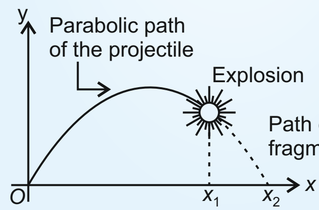
- Total mass of system of particles times the acceleration of its centre of mass is vector sum of all forces acting on system of particles.

$$M\vec{A} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + F_n$$

$$M\vec{A} = \vec{F}_{ext}$$

$$\vec{A} = \frac{\vec{F}_{ext}}{M} = \frac{\text{Total external force}}{\text{Total mass of system}} = \frac{\Sigma m_i\vec{a}_i}{M}$$

- Centre of mass of the system of particles moves as if all mass of a system was concentrated at centre of mass and all the external forces were applied at that point.
- A projectile following parabolic path explodes into fragments in mid air. The forces leading to explosion are internal, they contribute nothing to motion of COM. Total external force gravity acting on body is same before and after explosion. The COM under influence of external forces continue along same parabolic trajectory as it would have followed without explosion.



4 LINEAR MOMENTUM OF SYSTEM OF PARTICLES

- Velocity of COM for a system of n particles

$$\vec{P} = \vec{p}_1 + \vec{P}_2 + ...+ \vec{p}_n = m_1\vec{v}_1 + m_2\vec{v}_2 + ...+ m_n\vec{v}_n$$

$$\vec{V} = \frac{m_1\vec{v}_1 + m_1\vec{v}_2 + ... + m_n\vec{v}_n}{M}$$

This is the velocity of centre of mass

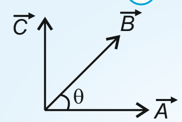
- Total linear momentum of system of particles is equal to the product of total mass of system and velocity of its centre of mass.
- When total external force acting on a system of particles is zero, total linear momentum of system is constant. The velocity of centre of mass remains constant.

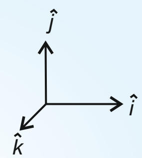
$$\vec{P} = m\vec{v}$$

$$\text{if } \vec{F}_{ext} \Rightarrow \frac{d\vec{P}}{dt} = 0 \quad \boxed{P = \text{constant}}$$

- If centre of mass was initially at rest, for no external force, centre of mass will remain at rest.

5 CROSS PRODUCT





- $\vec{C} = \vec{A} \times \vec{B}$

$|\vec{C}| = |\vec{A}||\vec{B}|\sin\theta$

θ is angle between $\vec{A}$ and $\vec{B}$

- Properties

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

$\vec{A} \times \vec{A} = \vec{0}$

$\hat{i} \times \hat{i} = \vec{0}$

$\hat{j} \times \hat{j} = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

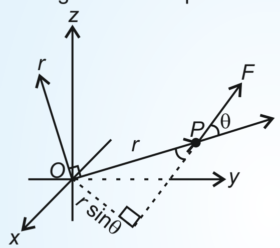
$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{k} = -\hat{i}$

6 MOMENT OF FORCE (TORQUE)

- Analogue of force in case of rotational motion is torque, which is turning effect of a force.
- $\vec{\tau} = \vec{r} \times \vec{F}$ when force acts on a particle whose position vector w.r.t. origin is $\vec{r}$.
- This is a vector quantity having SI units N m.
- Magnitude of torque



$\tau = r F\sin\theta$

$\tau = (r\sin\theta) \times F = r_\perp F$

$\tau = rF\sin\theta = rF_\perp$

$r_\perp = r\sin\theta$ = perpendicular distance of line of action of force from origin (axis of rotation) and $\vec{F}_\perp$ is component of $\vec{F}$ Perpendicular to $\vec{r}$.

- If direction of $\vec{r}$ and $\vec{F}$ are reversed, the direction of moment of force remains same.

- Couple : A pair of equal and opposite forces with different lines of action is known as a couple. A couple produces rotation without translation example : opening a bottle.

Bottle

8 EQUILIBRIUM OF RIGID BODY

A rigid body is said to be in mechanical equilibrium if both its linear momentum and angular momentum are not changing with time or equivalently, the body has neither linear acceleration nor angular acceleration.

- Vector sum of forces on rigid body is zero $\sum \vec{F}_i = 0$
- Vector sum of torques on rigid body is zero. $\sum \vec{\tau}_i = 0$
- Rotational equilibrium condition is independent of location of origin about which torques are taken.
- A body may be in partial equilibrium i.e. rotational equilibrium but not transnational.

7 ANGULAR MOMENTUM

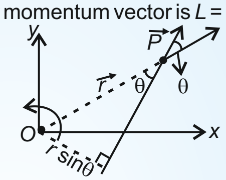
- It is referred as moment of linear momentum. For a particle,

$\vec{L} = \vec{r} \times \vec{P}$

The magnitude of angular momentum vector is $L = rp\sin\theta$

$L = r \times P\sin\theta = r \times P_\perp$

$L = r\sin\theta \times P = r_\perp \times P$



$r_\perp = (r\sin\theta)$ = is perpendicular distance of directional line of $\vec{P}$ from origin and

$P_\perp$ = component of P in the direction perpendicular to $\vec{r}$

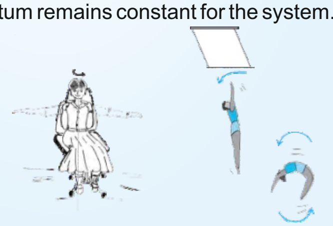
- Angular momentum will be zero when $P = 0$ or particle is at origin or or line of P passes through origin.
- **Angular Momentum Conservation Law**

$\vec{\tau} = \frac{d\vec{L}}{dt}$ (time rate of change of angular momentum of a system of particles is equal to torque acting on it.)

- If total external torque on a system of particles is zero, total angular momentum remains constant for the system.

$\vec{\tau} = 0 \rightarrow \frac{d\vec{L}}{dt} = 0$

$\vec{L}$ = constant



9 MOMENT OF INERTIA : MOI

- Analogue of mass, in rotational motion is rotational inertia also called moment of inertia.
- This is a characteristics of rigid body and the axis about which it rotates. It depends on distribution of mass and position of axis of rotation.
- This parameter is independent of magnitude of angular velocity of body, For a system of particles moment of inertia is given by $I = \sum_{i=1}^{n} m_i r_i^2$

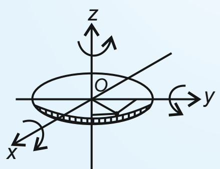
MOMENT OF INERTIA OF DIFFERENT RIGID BODIES
(Regular Shaped)

Body	Axis	I
Thin circular ring, radius R	Perpendicular to plane, at centre	MR^2
Thin circular ring, radius R	Diameter	$MR^2/2$
Thin rod, length L	Perpendicular to rod, at mid point	$ML^2/12$
Circular disc, radius R	Perpendicular to disc at centre	$MR^2/2$
Circular disc, radius R	Diameter	$MR^2/4$
Hollow cylinder, radius R	Axis of cylinder	MR^2
Solid cylinder, radius R	Axis of cylinder	$MR^2/2$
Solid sphere, radius R	Diameter	$2MR^2/5$
Hollow sphere, radius R	Diameter	$2MR^2/3$

THEOREMS OF MOI

Theorem of perpendicular axes

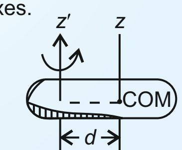
- Theorem is applicable to bodies whose thickness is small compared to other dimensions. (Planar body)
- MOI of a planar body about an axis perpendicular to its plane is equal to the sum of its MOI about two perpendicular axes concurrent with perpendicular axis and lying in plane of body.



$I_z = I_x + I_y$

Theorem of parallel axes

- The theorem is applicable to body irrespective of any shape.
- MOI of a body about any axis is equal to the sum of MOI of the body about a parallel axis passing through its COM and the product of its mass and the square of distance between the two parallel axes.



$I_{z'} = I_z + md^2$

10 TRANSNATIONAL AND ROTATIONAL MOTION ANALOGY

Linear motion	Rotation about a fixed Axis
Displacement x	Angular displacement θ
Velocity $v = \frac{dx}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
Mass m	Moment of inertia I
Force $F = ma$	Torque $\tau = I\alpha$
Work $dW = Fds$	Work $= dW = \tau d\theta$
Kinetic energy $k = \frac{mv^2}{2}$	Kinetic energy $k = \frac{I\omega^2}{2}$
Power $P = \vec{F}.\vec{V}$	Power $P = \vec{\tau}.\vec{\omega}$
Linear momentum $P = mV$	Angular momentum $L = I\omega$
$\vec{F} = \frac{d\vec{p}}{dt}$	$\vec{\tau} = \frac{d\vec{L}}{dt}$

12 KINETIC ENERGY OF TRANSLATING AND ROTATING BODIES:

- K.E of translation + K.E of rotational motion

$$KE = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I\omega^2$$

where $I = MK^2$, K is corresponding radius of gyration

Radius of Gyration : Distance from axis of rotation of a point mass whose mass is equal to mass of whole body and whose moment of inertia is equal to moment of inertia of body about the axis.

- **Kinetic Energy In Case Of Pure Rolling Motion**

$$V_{cm} = R\omega_{cm}$$

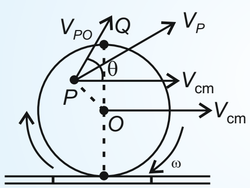
$$KE = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}MK^2\left(\frac{V_{cm}^2}{R^2}\right)$$

$$KE = \frac{1}{2}MV_{cm}^2\left[1 + \frac{K^2}{R^2}\right]$$

This formula can be used to all rolling bodies like ring, disc, cylinder sphere.

11 ROLLING MOTION

- All wheels used in transportation have rolling motion.
- It is combination of rotation and translation with axis moving.
- When disc rolls without slipping, At any instant of time bottom of disc which is in contact with surface is at rest with respect to surface.



$$\vec{V}_P = \vec{V}_{PO} + \vec{V}_{cm}$$

$$|V_P| = [(V_{PO})^2 + V_{cm}^2 + 2V_{PO}V_{cm}\cos\theta]^{1/2}$$

- In pure rolling with out slipping $\Rightarrow v = R\omega$
- Top of a rolling body has magnitude of velocity

$V_Q = V_{cm} + \omega_{cm}R = V_{cm} + V_{cm} = 2V_{cm}$, Bottom is at rest w.r.t. surface

13 KINEMATICS OF ROLLING BODIES DOWN ROUGH INCLINE PLANE

We apply conservation of mechanical energy to rolling bodies as Rolling friction performs no work.

$$\Delta P.E. = \Delta K.E.$$

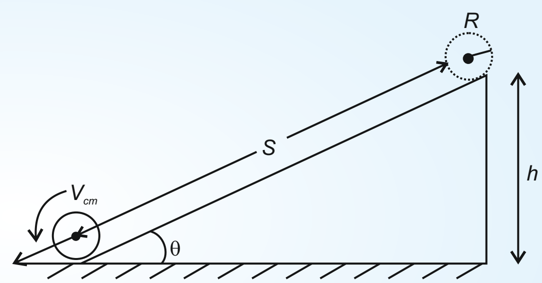
$$mgh = \frac{1}{2}mV_{cm}^2\left[1 + \frac{K^2}{R^2}\right]$$

Velocity at bottom $V_{cm} = \sqrt{\dfrac{2gh}{1 + \frac{K^2}{R^2}}}$

Acceleration of COM ; $a = \dfrac{g\sin\theta}{1 + (K/R)^2}$

Minimum coefficient of friction required for pure rolling $\mu = \left(\dfrac{K^2}{R^2 + K^2}\right)\tan\theta$

Time to reach the bottom $= \dfrac{1}{\sin\theta} \times \sqrt{\dfrac{2h}{g} \times \left(\dfrac{K^2}{R^2} + 1\right)}$



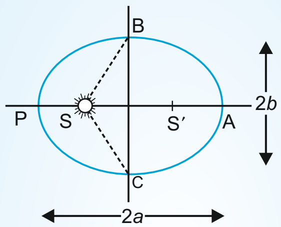
Gravitation

Chapter 8

1 KEPLER'S LAWS OF PLANETARY MOTION

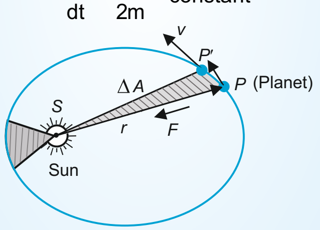
Law of Orbits

Every planet revolves around the sun in an elliptical orbit and the sun in situated at one of its foci.



Law of Areas

The areal velocity of the planet around the sun is constant i.e. $\frac{dA}{dt} = \frac{L}{2m}$ = constant



Law of Periods

The square of the time period of revolution of a planet is directly proportional to the cube of semi major axis length of the elliptical orbit *i.e.* $T^2 \propto a^3$

2 NEWTON'S LAW OF GRAVITATION

- The Gravitational force (F) between two bodies is directly proportional to product of masses and inversely proportional to square of distance between them.

$$\vec{F} = \frac{-Gm_1m_2}{r^2}\hat{r}$$

Characteristics of Gravitational Force

- It is alway attractive
- It is independent of the medium
- It is a conservative and central force
- It has infinite range

Superposition Principle

The Gravitational force on a point mass m_1 is the vector sum of the gravitational forces exerted by m_2, m_3 ....

i.e. $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ...$

4 VARIATION OF ACCELERATION DUE TO GRAVITY (g)

Due to Altitude (h)

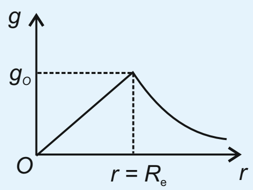
The value of g goes on decreasing with height (h)

$$g_h = \frac{GM_e}{(R_e+h)^2}$$

Due to Depth (d)

The value of g decreases with depth

$$g_d = g\left(1 - \frac{d}{R_e}\right)$$



3 ACCELERATION DUE TO GRAVITY

- For a body falling freely under gravity, the acceleration of body is called acceleration due to gravity
- $g = \frac{GM_e}{R_e^2} = \frac{4}{3}\pi G\rho R_e$

Where G = Gravitational constant

$\rho \to$ Average density of earth

$M_e \to$ Mass of earth

$R_e \to$ Radius of earth

5 GRAVITATIONAL POTENTIAL ENERGY

- The work done in bringing a body from infinity to a point in the gravitational field is gravitational potential energy

For two point mass system

$$U = -\frac{Gm_1m_2}{r}$$

Gravitational Potential due to a point mass

It is the work done in bringing a unit mass from infinity to a point in the gravitational field. $V = -\frac{Gm}{r}$

6 ESCAPE SPEED

- The minium speed of projection of a body from surface of earth so that it just crosses the gravitational field of earth

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e} = \left(\sqrt{\frac{8\pi G\rho}{3}}\right)R_e$$

It is independent of angle of projection.

- Escape velocity from moon is about 5 times smaller than earth.

7 EARTH'S SATELLITE

Orbital Speed of Satellite

- The speed required to put satellite into a given circular orbit

$$v_0 = \sqrt{\frac{GM_e}{R_e+h}} = R_e\sqrt{\frac{g}{R_e+h}}$$

- For satellite very close to earth orbital speed

$$v_0 = \sqrt{\frac{GM_e}{R_e}} = \sqrt{gR_e} = \frac{v_e}{\sqrt{2}}$$

Time Period of Satellite

- $T = \frac{2\pi}{\sqrt{GM_e}}(R_e+h)^{3/2} = \frac{2\pi}{R_e}\sqrt{\frac{(R_e+h)^3}{g}}$
- For satellite very close to earth's surface

$$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6 \text{ min}$$

Energy of Satellite

- Kinetic energy $K = \frac{GM_em}{2(R_e+h)}$
- Potential energy $U = -\frac{GM_em}{(R_e+h)}$
- Total energy $(E) = K + U$

$$= -\frac{GM_em}{2(R_e+h)}$$

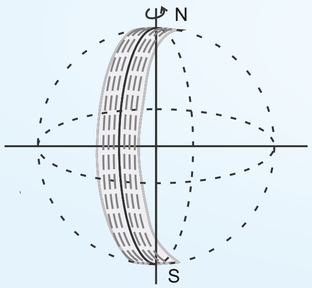
- Binding energy $(BE) = -E$

$$= +\frac{GM_em}{2(R_e+h)}$$

8 TYPES OF SATELLITES

Polar Satellite

- Revolves in polar orbit around the earth
- Height is approximately 500 to 800 km
- Time period is nearly 100 min
- Used in military spying, weather forecasting, meteorology etc.



Geostationary Satellite

- Time period is 24 h.
- Height is approximately 35800 km.
- Have same angular speed and sense of rotation as of earth
- Used for satellite communication, GPS
- INSAT is group of Geostationary satellites sent up by India.

9 WEIGHTLESSNESS

An Astronaut experiences weight- lessness in a space satellite. This is not because the gravitational force is small at that location in space. It is because both the astronaut and every part of satellite has an acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position.

Chapter 9
Mechanical Properties of Solids

1 ELASTICITY AND PLASTICITY

- **Elasticity :** Property of a body to regain its original shape and size, on removing the deforming force
- **Plasticity:** The inability of a body to regain its original size and shape on the removal of the deforming forces

2 STRESS AND STRAIN

- Stress = $\frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$ unit : $N\,m^{-2}$

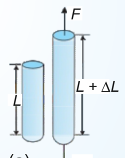
(a) **Longitudinal stress :**
Tensile stress : When a cylinder is stretched by two equal forces normal to its cross-sectional area the restoring force per unit area is called Tensile stress.
Compressive stress : If the cylinder is compressed under the action of applied forces, the restoring force per unit area is called compressive stress.

(b) **Tagential stress (or shear stress) :** The restoring force per unit area developed due to applied tangential force is called tangential or shearing stress.

(c) **Hydraulic stress :** It is the restoring force per unit area. When a body under high pressure is compressed uniformly on all sides, the magnitude is equal to hydraulic pressure.

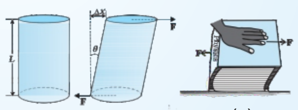
- Strain = $\frac{\text{Change in dimension}}{\text{Original dimension}}$ (No unit)

(a) Longitudinal strain = $\frac{\Delta L}{L}$

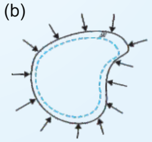


(a)

(b) Shear strain = $\frac{\Delta x}{L} = \tan\theta$


(b) (c)

(c) Volume strain = $\frac{\Delta V}{V}$


(d)

3 HOOKE'S LAW

Stress ∝ strain

Stress = k strain

k = modulus of elasticity

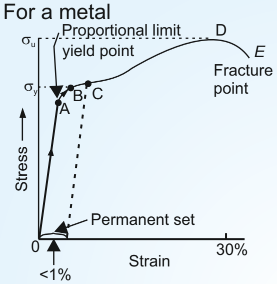
6 POISSON'S RATIO

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{(\Delta d/d)}{(\Delta L/L)}$$

Δd : Contraction in diameter of stretched wire.

4 STRESS - STRAIN CURVE

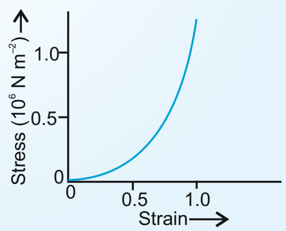
- For a metal



O to A : linear curve(Hooke's Law)
A : Proportional limit
B : Yield point(elastic limit) → Corresponding stress is yield strength(σ_y)
D : Ultimate tensile strength (σ_u)
E : Fracture point

- Material is brittle if D and E are close and ductile if D and E are far apart
- For an elastomer

Very large elastic region, even if material does not obey Hooke's law and there is no well defined plastic region.



5 VARIOUS OF MODULUS OF ELASTICITY

(a) Young's Modulus

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{FL}{A\Delta L}$$

(b) Bulk Modulus

$$B = \frac{\text{hydraulic stress}}{\text{volume strain}}$$

$$= -\frac{p}{(\Delta v/v)}$$

Compressibility $k = \frac{1}{B}$

(c) Shear modulus or modulus of rigidity

$$G = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{FL}{A\Delta x}$$

For most materials, $G = \frac{Y}{3}$

7 ELASTIC POTENTIAL ENERGY (IN A STRETCHED WIRE)

$$U = \frac{1}{2}\, YA \times \frac{\Delta L^2}{L} = \frac{1}{2} F\Delta L$$

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

Elastic potential energy per unit volume

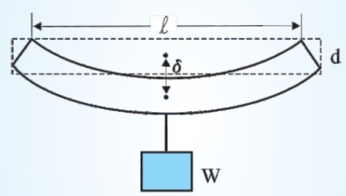
$$u = \frac{1}{2}\,\text{stress} \times \text{strain} = \frac{1}{2}\,\sigma\,\varepsilon$$

8 APPLICATIONS OF ELASTIC BEHAVIOUR OF MATERIALS

- Minimum area of cross - section of wire of crane

$$A = \frac{Mg}{\sigma_y}$$

- Designing beams for bridges



$$\delta = \frac{Wl^3}{(4bd^3Y)}$$

$\delta \propto d^{-3}$

So I shaped beam is preferred

- Maximum height of a mountain

$h = \frac{E}{\rho g}$, E is elastic limit

Mechanical Properties of Fluids

10 Chapter

1 PRESSURE

- Average pressure is defined as the normal force acting per unit area

$$P_{av} = \frac{F}{A}$$

$$P = \lim_{\Delta A \to 0}\left(\frac{\Delta F}{\Delta A}\right)$$, It is a scalar quantity.

2 VARIATION OF PRESSURE WITH DEPTH

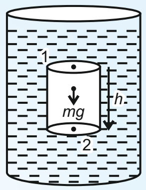
Pressure difference

$P_2 - P_1 = \rho gh$

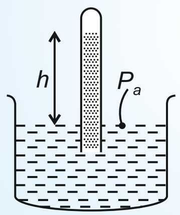
If point 1 is at free surface of liquid

then $P_2 = P_a + \rho gh$

$P - P_a = \rho gh$ (called gauge pressure)

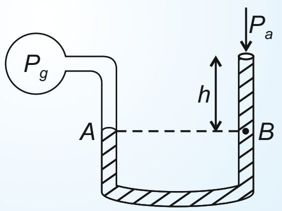


- **Instruments used to measure pressure**



Mercury barometer used to measure atmospheric pressure

$P_a = \rho gh$



The open tube manometer, used to measure pressure of gas

$P_g = P_a + \rho gh$

3 PASCAL'S LAW

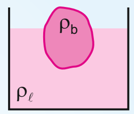
- When ever an external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.

Devices based on Pascal's law

(i) Hydraulic lift (ii) Hydraulic brakes

4 ARCHIMEDES PRINCIPLE

- Loss of weight of a body submerged (partially or completely) in a fluid is equal to the weight of the fluid displaced.



Weight of fluid displaced = $\rho_\ell V_s g = F_B$

If $\rho_b < \rho_\ell$; then body will float

If $\rho_b = \rho_\ell$; body will just float with fully submerged

If $\rho_b > \rho_\ell$; then body will sink

- **Law of Floatation**

For Floating object $\frac{V_s}{V_b} = \frac{\rho_b}{\rho_\ell}$

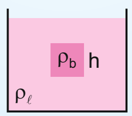
Fraction of vol. submerged = ratio of density of body and fluid

- **Buoyant Force**

Buoyant force is equal to weight of the fluid displaced.

Buoyant force depends on g_{eff}

Buoyant force acts opposite to g_{eff}



5 STREAMLINE FLOW

- The flow is said to be steady if at any given point, the velocity of each passing fluid particle remains constant in time. The path taken by fluid particle under steady flow called streamline.
- **Equation of continuity:** In stream line flow, mass of liquid coming out equals to the mass of liquid flowing in

$A_1v_1 = A_2v_2$

It is based on conservation of mass.

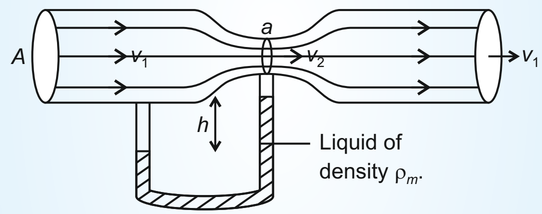
6 BERNOULLI'S EQUATION

- It states that for a steady flow of an ideal fluid, the sum of pressure energy per unit volume (P), kinetic energy per unit volume and potential energy per unit volume remains constant.

$P + 1/2\,\rho v^2 + \rho gh = \text{constant}$

- Phenomenons associated: Heart attack, magnus effect and aerofoil (lift of aircraft)
- **Venturi-meter:** It is a device used to measure the flow speed of incompressible fluid.

$$v_1 = \sqrt{\frac{2\rho_m gh}{\rho}}\left[\left(\frac{A}{a}\right)^2 - 1\right]^{-1/2}$$

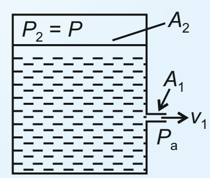


- **Speed of efflux: Torricelli's Law**

$A_1v_1 = A_2v_2$, if $v_2 << v_1$

$$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$$

- When $P >> P_a$ and $2gh$ may be ignored.
- On the other hand tank is open to atmosphere, then $P = P_a$



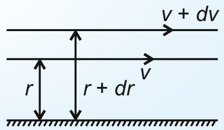
$v_1 = \sqrt{2gh}$

7 VISCOSITY

- The property of fluid due to which it opposes relative motion between its different layers in steady flow called viscosity.
- Tangential force between the layer

$F = -\eta A \frac{dv}{dr}$

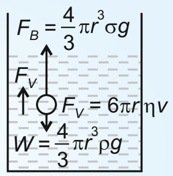
η = coefficient of viscosity



- $\eta = \frac{\text{Shearing stress}}{\text{Shear strain rate}}$
- SI unit is poiseiulle (ρ_ℓ).

8 STOKE'S LAW

- The viscous force acting on a spherical body of radius a. $F = -6\pi\eta a v$



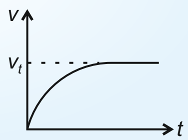
- **Terminal velocity:**

$v_t = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

ρ = density of sphere material

σ = density of fluid

- **Variation of Velocity with Time**



9 REYNOLDS NUMBER (R_e):

$R_e = \frac{\text{Inertial force}}{\text{Viscous force}}$ (dimensionless)

$R_e = \frac{\rho v d}{\eta}$

Where v = velocity of liquid

ρ = density of liquid

- The flow is turbulent for $R_e > 2000$.
- Flow is unsteady for R_e between 1000 and 2000.
- Flow is streamline for R_e less than 1000.
- Critical Reynold number is one at which turbulence sets.
- Reynold number helps study nature of fluid flow.
- Turbulence dissipates kinetic energy in the form of heat.

10 SURFACE TENSION

- **Surface:** It is the thickness of few molecular size.
- **Surface Tension:** The property of liquid by which the free surface of liquid at rest tends to have minimum surface area called surface tension.

  Surface tension can be defined as the force per unit length on imaginary line drawn at the surface of liquid

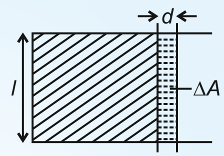
  $S = \frac{F}{l}$

- Surface tension of a liquid falls with temperature.
- **Surface energy:** Molecules on the surface of liquid have some extra potential energy in comparison to molecules in the interior. A liquid thus tends to have minimum surface area.

11 SURFACE ENERGY AND SURFACE TENSION

- Work done in increasing surface area.

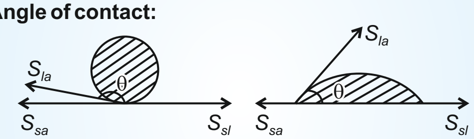
  $W = S(\Delta A_{eff})$



  - Thin film, liquid bubble have two surfaces so,

  $\Delta A_{eff} = 2\Delta A_{Geo}$.

- **Angle of contact:**

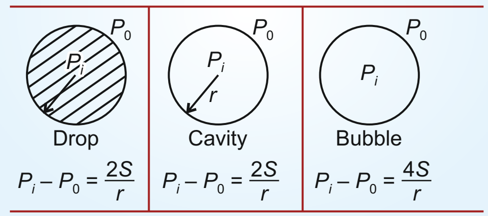


- At the point of contact, the angle between tangent planes drawn at the surface of liquid and at surface of solid inside liquid called angle of contact.
  - If $\theta < 90° \rightarrow$ Surface will be concave, liquid stick to solid and rise in capillary.
  - If $\theta > 90° \rightarrow$ Surface will be convex, liquid does not stick to solid and fall in capillary.
  - If $\theta = 90° \rightarrow$ Surface will be plane, liquid does not stick to solid neither rise nor fall in capillary.
  - Water forms droplets over a lotus leaf while spreads over a clean plastic plate.

12 DROPS AND BUBBLES

- $P_{inside} > P_{outside}$ (For liquid-gas interface, the convex side has lower pressure than on concave side.)
  - Liquid drop, air bubble in water have one surfaces so,

  $\Delta A_{eff} = \Delta A_{Geo}$

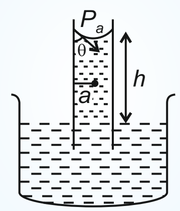


13 CAPILLARITY

- When a capillary tube is dipped in any liquid then liquid either rise or fall inside the capillary tube.
- Height of liquid column rise or fall inside a capillary tube is

$h = \frac{2S\cos\theta}{a\rho g}$

$h \propto \frac{1}{a}$



- In a tube of insufficient length, liquid will rise to the top of capillary, increase radius of curvature and stay there. Never comes out in the form of fountain.
- This is consequence of pressure difference across a curved liquid air interface a well known effect that water rises up in narrow tube inspite of gravity.

Thermal Properties of Matter

11
Chapter

1 TEMPERATURE

- Temperature is a relative measure of hotness or coldness.
- Heat transfer takes place between system and surrounding medium until they are at same temperature.
- Measure of temperature is obtained using a thermometer.
- Some properties of material change with temperature to be used as basis of constructing thermometer.
- For standard scale a fixed reference point is taken.
- A relationship for conversion between Fahrenheit and Celsius temperature scale is

$$\frac{t_F - 32}{180} = \frac{t_C}{100}$$

- A temperature –273.15°C is designated as absolute zero. This is foundation of Kelvin temperature scale.
- Size of unit of Kelvin and Celsius temperature scales is the same. Relation between scales is

$$T_K = t_C + 273.15$$

2 HEAT

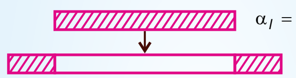
A form of energy transferred between two or more systems by virtue of temperature difference.

Thermal Expansion

- A change in temperature of a body causes change in its dimensions.

Three types of expansion

1. Linear Expansion



$$\alpha_l = \frac{\Delta l}{l} \times \frac{1}{\Delta T}$$

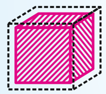
2. Area Expansion

$$\alpha_a = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$$

$\alpha_a = 2\alpha_l$

For anisotropic solid $\alpha_a = \alpha_{l_1} + \alpha_{l_2}$

3. Volume Expansion



$$\alpha_V = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

- α_V is constant only at high temperature
- Pyrex glass and invar has low α_V.
- Alcohol has high volume expansion coefficient than mercury.
- $\alpha_V = \frac{1}{T}$ for ideal gases

$(\alpha_V)_{gases} > (\alpha_V)_{liquid} > (\alpha_V)_{solids}$

- When a solid rod has its ends rigidly fixed, it results in thermal stress in material which is proportional to temperature change.

Thermal Stress = $Y \cdot \alpha_l \cdot \Delta T$

3 CALORIMETRY

- Heat lost by a part at higher temperature is equal to heat gained by the part at lower temperature.
- Calorimetry means measurement of heat.
- A device in which heat measurement can be done is called a calorimeter.

4 HEAT CAPACITY

The change in temperature of a substance, when a given quantity of heat is absorbed or rejected is characterised by a quantity called heat capacity.

$$S = \frac{\Delta Q}{\Delta T}$$

Specific heat capacity

This is unique value of heat absorbed or given off, to change unit mass of it by one unit temperature change.

$$s = \frac{S}{m} = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

Molar specific heat

If the amount of substance is specified in terms of moles
we define heat capacity per mole

$$C = \frac{S}{\mu} = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right) \quad \text{J mol}^{-1}\ \text{K}^{-1}$$

For gases two molar specific heat capacities

- Molar specific heat capacity at constant pressure C_P
- Molar specific heat capacity at constant volume C_V

$C_P - C_V = R$ (for ideal gases)

5 CHANGE OF STATE

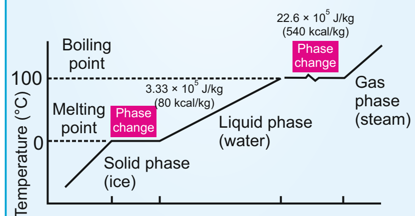
- Change of state from solid to liquid is called melting or fusion.
- Change of state from liquid to vapour is called vaporisation
- The temperature at which the liquid and vapour states of substance coexist is called its Boiling point.
- Boiling point increases with increase in pressure and vice versa.
- The change from solid state to vapour state without passing through the liquid state is called sublimation and substance is said to sublime.

Latent heat

- Amount of heat transferred during change of state of substance is called its latent heat.

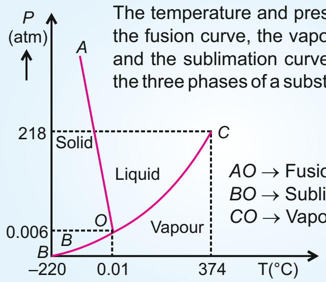
$$L = \frac{\Delta Q}{M} \ \text{J kg}^{-1}$$

- L depends on pressure.
- Solid-liquid state change → Latent heat of fusion (L_f)
- Liquid-gas state change → Latent heat of vaporisation (L_v)



6 TRIPLE POINT

The temperature and pressure at which the fusion curve, the vaporisation curve and the sublimation curve meet and all the three phases of a substance coexist



AO → Fusion curve
BO → Sublimation curve
CO → Vaporisation curve

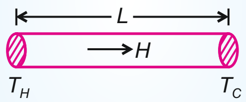
Pressure-temperature phase diagrams for water

HEAT TRANSFER MODES

7 CONDUCTION

- It is a mechanism of heat transfer between adjacent parts of a body due to temperature difference.
- At steady state, the temperature of bar, through which heat is flowing, decreases with distance, and heat starts flowing at a constant rate.
- The rate of flow of heat

$$H = KA\left(\frac{T_H - T_C}{L}\right)$$



T_H = Hot end Temperature

T_C = Cold end Temperature

L = Length of rod

A = Cross -section of rod

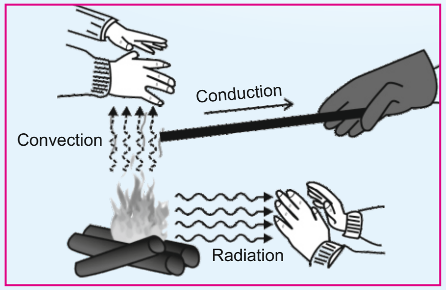
K is called thermal conductivity of material.

- Greater value of *K* for a material, more rapidly will it conduct heat. Its SI units are $Wm^{-1} K^{-1}$

8 CONVECTION

In convection mode heat transfer by actual motion of matter occurs.

- Convection occurs in fluids only.
- Two types of convections are
  1. Natural convection
  2. Forced convection
- Trade winds is example of natural convection in which gravity plays an important role.
- In forced convection material is forced to move by a pump. Human circulatory system, cooling system of automobile engine are forced convection method.



9 RADIATION

- This heat transfer mechanism needs no medium.
- Energy transferred by waves is called radiant energy.
- Heat transferred from sun to earth is by radiation.
- Radiations emitted by hot bodies are called thermal radiations.

LAWS OF RADIATIONS

10 Wien's Displacement Law

Wavelength for which radiation energy is maximum decreases with increasing temperature.

$$\lambda_m T = \text{constant}$$

Value of constant (Wien's constant) is 2.9×10^{-3} m K.

- This law is used to measure surface temperature of celestial bodies like stars, moon and sun.

11 Stefan-Boltzmann's Law

For a black body which is perfect radiator energy emitted per unit time is given as

$$H = A\sigma T^4$$

A is area, *T* is absolute temperature of body, σ is called Stefan Boltzmann's constant

$\sigma = 5.67 \times 10^{-8}\ Wm^{-2} K^{-4}$

- But if a body is surrounded by surroundings at temperature T_s.

For perfect radiator net rate of heat radiated

$$H = \sigma A(T^4 - T_s^4)$$

- For body with emissivity *e* modified relation is

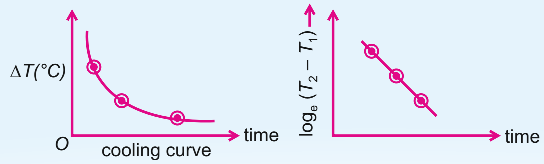
$$H = e\sigma A(T^4 - T_s^4) \quad 0 < e \le 1$$

12 Newton's law of cooling

Rate of heat loss of a body is directly proportional to difference of temperature of body and surroundings.

- This law holds for small temperature difference only.

$$-\frac{dQ}{dt} = k(T_B - T_S)$$



$$\log_e(T_2 - T_1) = -kt + C$$

Average method

$$\frac{\text{Change in temperature}}{\text{time}} = k\Delta T$$

$$\Delta T = (T_{av} - T_s)$$

13 Greenhouse effect:

The absorption of infrared waves by greenhouse gases such as CO_2, methane (CH_4) nitrous dude (N_2O) chlorofluorocarbon (CF_xCl_x) and ozone (O_3). Heating of atmosphere → More energy to earth → Warmer surface. Without Greenhouse effect temperature of earth would have been –18°C

Thermodynamics

12
Chapter

1 THERMODYNAMIC EQUILIBRIUM

- Temperature of a body is related to its average internal energy, not to kinetic energy of motion of centre of mass.
- Equilibrium in thermodynamics refer to situation when macroscopic variables defining thermodynamic state of system don't depend on time.

2 ZEROTH LAW OF THERMODYNAMICS

- Two systems in thermal equilibrium with third system separately are in thermal equilibrium with each other.
- If $T_A = T_C$ and $T_B = T_C$, then $T_A = T_B$
- Thermodynamic variable whose value is equal for two systems in thermal equilibrium is called temperature.

3 HEAT, INTERNAL ENERGY AND WORK

- Heat is energy transfer arising due to temperature difference between system and surroundings.
- Internal energy is simply the sum of kinetic energies and potential energies of the molecules in the frame of reference to which centre of mass of system is at rest.
- Internal energy depends on state of the system, not how the state was achieved.
- There are two ways to change internal energy of a thermodynamic system

  (1) To do work on system

  (2) Supply heat to system

  So heat and work are two modes of altering the state of a thermodynamic system and changing internal energy.
- Heat and work in Thermodynamics are not state variables.
- U is a state variable. ΔU depends only on initial and final states and not on path taken by gas to go from one to another.
- ΔQ and ΔW will depend on path taken to go from initial to final state.
- Work done during thermodynamic process

  $$\Delta W = \int_{V_1}^{V_2} P\,dV$$
- Area under the $P - V$ diagram with the volume axis gives the work done in thermodynamic process.

4 SPECIFIC HEAT CAPACITY

- Molar specific heat at constant volume.

  $$C_V = \left(\frac{\Delta Q}{\Delta T}\right)_V = \left(\frac{\Delta U}{\Delta T}\right)$$
- Molar specific heat at constant pressure

  $$C_P = \left(\frac{\Delta Q}{\Delta T}\right)_P = \left(\frac{\Delta U}{\Delta T}\right)_P + P\left(\frac{\Delta V}{\Delta T}\right)_P$$

  $$PV = RT \quad \therefore \quad P\left(\frac{\Delta V}{\Delta T}\right)_P = R$$
- $\boxed{C_P = C_V + R}$ (MAYER'S Equation)
- $$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$$

  $\boxed{C_P = \gamma \times C_V}$

5 FIRST LAW OF THERMODYNAMICS

- $\Delta Q = \Delta U + \Delta W$ (Energy conservation law)

  ΔQ = heat supplied to system by the surrounding

  ΔW = work done by the system on the surrounding

  ΔU = Change in internal energy of a the system
- Heat supplied to system goes in partly to increase internal energy and rest in work on environment.
- This is simply the general law of conservation of energy applied to any system in which energy transfer is taken into account.
- $\Delta W = P\Delta V$

$\therefore$ $\Delta Q = \Delta U + P\Delta V$

8 THERMODYNAMIC PROCESSES

- A thermodynamic process is an activity where a thermodynamic system is taken from one equilibrium state to another.
- Reversible process
- Irreversible process
- Cyclic process

6 THERMODYNAMIC STATE VARIABLES

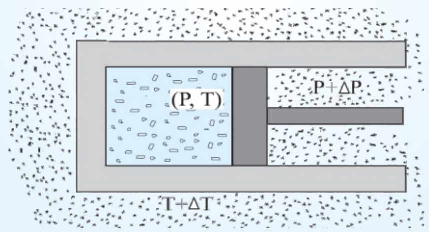
- Thermodynamic state variables describe equilibrium state of system. These state variables are not necessarily independent.
- The connection among state variables is called equation of state.
- Equilibrium state of thermodynamic system is described by state variables. The value of state variable depends on particular state not by the path used to arrive that state. Pressure, volume, temperature and mass are state variable. Heat and work are not state variables.
- For an ideal gas, equation of state is $\boxed{PV = \mu RT}$
- Thermodynamic state variables are of two types

  (1) Extensive

  (2) Intensive
- Extensive variables indicates size of system.
- Internal energy, volume and mass are extensive variables. But pressure, temperature and density are intensive variables.

7 REVERSIBLE AND IRREVERSIBLE PROCESS

- Spontaneous processes in nature are irreversible.
- A process is reversible if the process can be turned back such that both the system and surrounding return to their original states with no any other change anywhere else in universe.
- A quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is a reversible process.
- A quasi-static process is an infinitely slow process such that system remains in thermal and mechanical equilibrium with surroundings throughout. In this process pressure and temperature of the environment can differ from those of system only infinitesimally.



No accelerated motion of Piston

9 ISOTHERMAL PROCESS

- For isothermal process Temperature during the process should be constant

  $PV =$ constant
- So pressure of given mass of a gas varies inversely as its volume.
- **Work done in isothermal process.**

  If a system of ideal gas at temperature T goes from (P_1, V_1) to (P_2, V_2) equilibrium state, then work done

  $$W = \mu RT \ln\left(\frac{V_2}{V_1}\right) = \mu RT \ln\left(\frac{P_1}{P_2}\right)$$
- Here $\Delta T = 0 \quad \therefore \Delta U = 0$

  $$\Delta Q = \Delta W = \mu RT \ln\left(\frac{V_2}{V_1}\right)$$

10 ADIABATIC PROCESS

- In adiabatic process heat interaction between system and surrounding is zero. *i.e.* $\Delta Q = 0$
- $PV^{\gamma} =$ constant

  Where γ = ratio of molar specific heats at constant pressure and at constant volume.
- System is insulated from surroundings and heat absorbed or released is zero.
- Work done by gas results in decrease in its internal energy.
- If system change from (P_1, V_1, T_1) to (P_2, V_2, T_2)

  $$\Delta W = \frac{\mu R(T_1 - T_2)}{\gamma - 1} = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)} \text{ where } \gamma = C_P/C_V$$
- If work done by gas $(W > 0)$, then, $T_2 < T_1$

11 ISOBARIC PROCESS

- For isobaric process pressure during the process should be constant

  $\frac{V}{T} =$ constant
- Work done in isobaric process

  $W = P(V_2 - V_1) = \mu R(T_2 - T_1)$
- Heat partly to do absorbed goes partly to increase internal energy and mechanical work.

  $\Delta Q = \Delta U + \Delta W$

  $\Delta U = \mu C_V \Delta T$, $\Delta Q = \mu C_P \Delta T$ and $\Delta W = \mu R \Delta T$

  $$\frac{\Delta W}{\Delta Q} = \frac{R}{C_P} = \frac{\gamma - 1}{\gamma} \text{ and } \frac{\Delta U}{\Delta Q} = \frac{C_V}{C_P} = \frac{1}{\gamma}$$

12 ISOCHORIC PROCESS

- For isochoric process volume during the process should be constant

  $\frac{P}{T} =$ constant
- Work done in isochoric process, $\Delta W = P\Delta V = 0$
- $\Delta Q = \Delta U + \Delta W$

  $\Delta Q = \Delta U$
- Heat absorbed by gas goes entirely to change its internal energy and its temperature.
- Change in internal energy is determined by specific heat at constant volume and temperature change.

13 CYCLIC PROCESS

- In any cyclic process system returns to initial state, $\Delta U = 0$
- Hence total heat absorbed equals the work done by the system, $\Delta Q = \Delta W$

15 REFRIGERATOR

- A refrigerator is the reverse of a heat engine. Working substance extracts heat from cold reservoir, some external work is done on system and heat is released to reservoir at high temperature.
- Coefficient of performance of refrigerator $= \frac{\text{heat extracted}}{\text{work input}}$

  $$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{(T_1 - T_2)} = \frac{1 - \eta}{\eta}$$
- Coefficient of performance for heat pump is

  $$\beta = \frac{Q_1}{W} = \frac{T_1}{T_1 - T_2} = \frac{1}{\eta}$$

16 SECOND LAW OF THERMODYNAMICS

- **Kelvin-Planck statement :** No process is possible whose sole result is absorption of heat from a reservoir and complete conversion of heat into work.
- **Clausius statement :** No process is possible whose sole result is transfer of heat from cold reservoir to hotter object.
- Two statements are completely equivalent.
- It shows that efficiency of a heat engine can never be unity so heat released to cold reservoir can never be made zero.
- Kelvin Planck and Clausius deny the perfect heat engine and refrigerator.

14 HEAT ENGINE

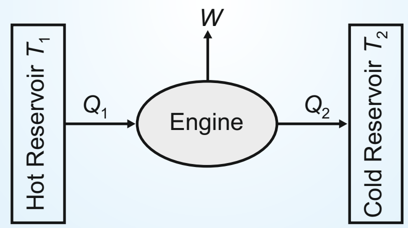
- Heat engine is a device in which a system undergoes a cyclic process resulting in conversion of heat in to the sink.
- Efficiency of the engine is

  $$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

  Q_1 = heat absorbed from source

  Q_2 = heat released to sink

  η = efficiency of heat engine

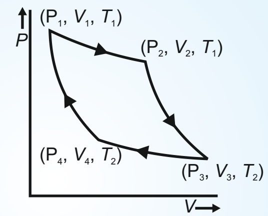


- Heat engine based on idealised reversible processes achieve the highest possible efficiency.

17 CARNOT ENGINE

- Carnot engine is a reversible engine operating between two temperatures T_1 and T_2. Carnot cycle consists of two isothermal and two adiabatic processes. Its efficiency is

  $$\eta = 1 - \frac{T_2}{T_1}$$



- Engine efficiency of Carnot engine does not depend on nature of working substance.

  Carnot Theorem: Any other engine working between temperature T_1 and T_2 can not have efficiency more than that of Carnot engine. The Carnot engine's efficiency is independent of nature of working substance.

  In Carnot cycle

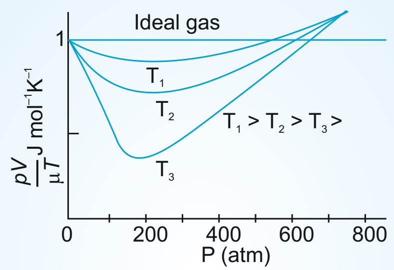
  $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ is universal relation and this relation can be used to design universal thermodynamic scale.

Kinetic Theory

13
Chapter

1 LAWS ASSOCIATED WITH KTG

- An ideal gas is only theoretical model of a gas. No real gas is truly ideal. Without interactions gas behaves like an ideal gas. At low pressure or high temperature, molecules are far apart and molecular interaction is negligible.



- **Boyle's Law :** Pressure of a given ideal gas is inversely proportional to its volume if temperature is kept constant.
- **Charle's Law :** Volume of given ideal gas is directly proportional to its temperature in kelvin if pressure is kept constant.
- **Gay Lussac's Law :** Pressure of an ideal gas is directly proportional to its absolute temperature if volume is kept constant.
- **Avogadro's Law :** Equal volume of all the gases under similar conditions of temperature and pressure contain equal number of molecules.

$$\frac{P_1V_1}{N_1T_1} = \frac{P_2V_2}{N_2T_2} = K_B$$

Ideal gas equation connecting the variables is

$$\boxed{PV = \mu RT = K_BNT} \quad P = \frac{\rho RT}{M_0}$$

- **Dalton's Law of Partial Pressure :** Total pressure of a mixture of non-reactive gases is the sum of their partial pressures.

2 AVERAGE PRESSURE OF GAS

$$\boxed{P = \frac{1}{3}n\, m\bar{v}^2 \text{ and } PV = \frac{1}{3}nV\, m\bar{v}^2}$$

$n \rightarrow$ Number density
$m \rightarrow$ Mass of molecule
$\bar{v}^2 \rightarrow$ Mean of squared speed

3 KINETIC INTERPRETATION OF TEMPERATURE

- **Average kinetic energy of molecule**

$$= \frac{1}{2}m\bar{v}^2 = \frac{3}{2}K_BT$$

$$V_{RMS} = (\bar{V}^2)^{\frac{1}{2}}$$

$$= \sqrt{\frac{3K_BT}{m}}$$

- In a mixture of gases at a given temperature, heavier molecule has lower average speed.
- Translational kinetic energy of gas

$$\boxed{E = \frac{3}{2}K_BNT} \text{ and } \boxed{PV = \frac{2}{3}E}, \frac{E}{N} = \frac{1}{2}m\bar{v}^2 = \frac{3}{2}K_BT$$

- Average KE is proportional to temperature.
- R.M.S. speed of gas molecule,

$$V_{RMS} = \sqrt{\frac{3RT}{M}}$$

- Most probable speed of molecule

$$= \sqrt{\frac{2RT}{M}}$$

- Average speed of gas molecule

$$= \sqrt{\frac{8RT}{\pi M}}$$

- This concept of maxwell energy distribution predict specific heat of gases theoretically.

LAW OF EQUIPARTITION OF ENERGY

- KTG is consistent with ideal gas equation.
- For a system in equilibrium at absolute temperature T, total energy is distributed equally in different modes of absorptions. Energy of each mode is equal to 1/2 K_BT.
- Each translational and rotational degree of freedom corresponds to one energy mode of absorption.

4 SPECIFIC HEAT CAPACITIES

- Specific heat capacity for solids = $3R$
- Specific heat capacity of water = $9R$
- C_V (monoatomic gas) = $\frac{3}{2}R$
- $\gamma = 1 + \frac{2}{f}$
- $\gamma = \frac{5}{3}$ (monoatomic)
- $\gamma = \frac{7}{5}$ (rigid diatomic)
- Polyatomic gases in general a polyatomic molecules has 3 translational, 3 rotational degree of freedom and a certain number (f) of vibrational modes. Then for one mole of gas

$$U = \left[\frac{3}{2}K_BT + \frac{3}{2}K_BT + fK_BT\right]N_A$$

$$C_V = (3 + f)R \qquad \gamma = \frac{(4+f)}{(3+f)}$$
$$C_P = (4 + f)R$$

- Each vibrational frequency has two modes of energy with corresponding energy equal to K_BT.
- Molecules of a monatomic gas have only translational degree of freedom.
- Molecules like CO even at moderate temperature has mode of vibration.
- Diatomic molecule, like a dumbell, has five degree of freedom.
- Polyatomic molecule has 3 transitional, 3 rotational and a degree of a certain number of vibrational modes.

5 MEAN FREE PATH

- Molecules of gas have rather large speeds of the order of speed of sound.
- Molecules of gas undergo collisions and their paths keep getting deflected.
- Average distance a molecule can travel without collision is called mean free path.
- Mean free path of gas molecule is related to number of molecules per unit volume and size of gas molecule.

$$\boxed{\lambda = \frac{1}{\sqrt{2}\pi nd^2}} \equiv \boxed{\lambda = \frac{K_BT}{\sqrt{2}\pi Pd^2}}$$

n : number density; d : diameter of molecules

- Mean free path in gases is of order of thousands of angstrom.
  P : Pressure of gas; T : Absolute temperature
  K_B : Boltzmann's Constant

Oscillations

14
Chapter

1 SPECIAL TYPES OF MOTIONS

- **Periodic Motion**

  A motion which repeats itself after regular intervals of time, (T) is periodic. Examples:
  - Motion a particle in circle with constant speed
  - Skipping
  - Spring block system
  - Simple pendulum
  - Motion of Earth around sun
  - Motion of needle of sewing machine
  - A boat tossing up and down in a lake
  - Piston of engine going back and forth can be periodic
- **Oscillatory Motion**

  Special type of periodic motion in which a particle moves to and fro about a fixed point. The force acting on the particle in a direction directed towards equilibrium position is called **restoring force**.
  - Every oscillatory motion is periodic but every periodic motion may not be oscillatory.
  - Back and Forth motion can be oscillatory or vibratory. When oscillations frequency is small we call it oscillatory, at high frequency we call it vibratory.

  Oscillations can be

  A. **Free oscillations**
  - When a system oscillates with its natural frequency the oscillations are called free oscillations.

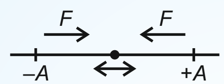
  B. **Damped oscillations**
  - If some external resisting force appears opposing restoring force, oscillatory amplitude starts decreasing with time.

  C. **Forced oscillations**
  - Forced oscillations are those in which damping is not allowed by applying an external time varying force, which compensates the effect of damping force acting on it.

2 SIMPLE HARMONIC MOTION

- Simple harmonic motion is an example of periodic oscillatory motion.
- Special type of oscillatory motion which satisfies following conditions.
  A. Oscillatory amplitude of particle is small.
  B. During oscillation, acceleration towards mean position, due to net restoring force, is directly proportion to displacement from mean position.
- Force displacement relation in S.H.M.
  $F = -ky$, where K is force constant (Force law in S.H.M.), y is displacement from mean position.
- Acceleration of particle

  $$a = \frac{F}{m} = -\left(\frac{K}{m}\right)y = -\omega^2 y$$



  $\therefore$ Acceleration and displacement are antiparallel

  $$\frac{d^2y}{dt^2} + \omega^2 y = 0\text{, here } \omega = \sqrt{\frac{K}{m}} \text{ (Angular frequency)}$$

  m is mass oscillating, K is force constant.
- **General equation for displacement in S.H.M.**

  $y = A\sin(\omega t + \phi)$ or $y = A\cos(\omega t \pm \phi)$

  $\omega = \frac{2\pi}{T} = 2\pi n$ is angular frequency and $(\omega t + \phi)$ is called phase, a time varying quantity.

  Here ϕ is called **epoch** or initial phase.

  A. If particle at $t = 0$ is at equilibrium position. $(y = 0)$

     $y = A\sin\omega t$

  B. If particle at $t = 0$ is at extreme right position $(y = A)$

     $y = A\cos\omega t$
- **Velocity of particle in SHM.**

  $$v_P = \frac{dy}{dt} = \omega A\cos(\omega t \pm \phi)$$

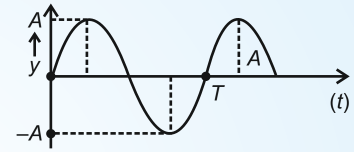
  If at $t = 0$ particle is at origin.

  $$v_P = \omega A\cos\omega t = \omega\sqrt{A^2 - y^2}$$
- **Acceleration of particle in SHM**

  $a_P = -\omega^2 A\sin\omega t$, at $t = 0$ particle is at mean position.
- Velocity displacement graph will be an ellipse $(\omega \neq 1)$ or a circle $(\omega = 1\text{ rad s}^{-1})$.
- The maximum velocity of particle executing SHM will be at mean position and at extremes speed becomes minimum (zero).
- Different graphs for a particle executing SHM

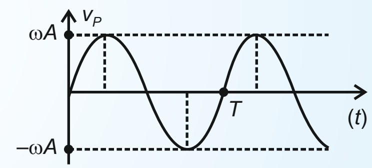
  (A) **Displacement - time graph**

  If at $t = 0$ particle is at mean position



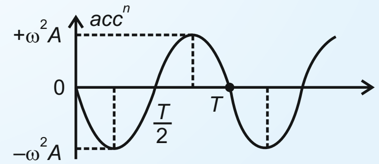
  $y = A\sin(\omega t)$

  (B) **Velocity - time graph**



  $v = A\omega\cos(\omega t)$

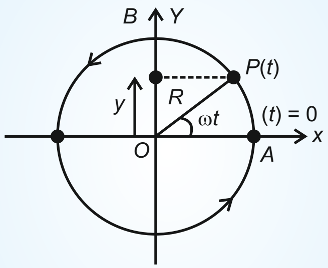
  (C) **Acceleration time graph**



  $a = -\omega^2 A\sin(\omega t)$
  - Velocity leads displacement by a phase of $(\pi/2)$ rad.
  - Acceleration leads velocity a phase of $\pi/2$ rad.

3 SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

- Projection of uniform circular motion on a diameter of the circle follows simple harmonic motion.



Particle position
$y = R \sin \omega t$ is SHM.

This is an equation of S.H.M. for particle displacement at any time.

4 SPECIAL PARAMETERS IN SHM

- Since particles speed is not constant ; from mean position to half of amplitude it takes half of time than to move from half of amplitude to extreme position.
- Minimum velocity in S.H.M. is $v_{min} = 0$ at extremes and maximum velocity at equilibrium position. $v_{max} = \omega A$
- Maximum acceleration of particle is at extreme positions $a_{max} = \omega^2 A$ and minimum (zero) is at equilibrium.
- Maximum force on particle is at extreme positions and zero at mean, in between it varies linearly always directed towards equilibrium.

$F_{max} = m\omega^2 A$ and F_{min} = zero

5 MECHANICAL ENERGY IN SIMPLE HARMONIC OSCILLATOR

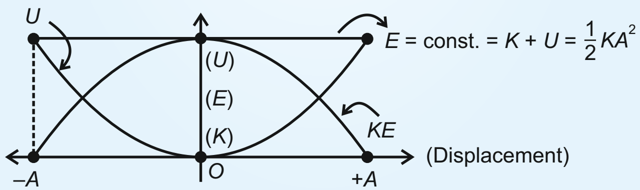
- Potential energy in SHM $U = \frac{1}{2}ky^2 + U_0$

  U_0 is generally taken zero at equilibrium.

  $F_{int} = -\frac{dU}{dy}$; instantaneous force on particle.
- Maximum potential energy occurs at extreme positions and minimum at mean position.
- Graph of potential energy versus displacement of particle will be parabolic, symmetric about y-axis.
- Kinetic energy of particle in S.H.M. varies directly as square of its velocity at any location.

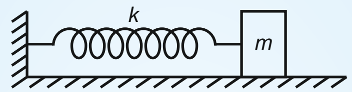
  $KE = \frac{1}{2}(m\omega^2)(A^2 - y^2) = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$
- Kinetic energy can not be negative. Potential energy increases at expense of KE and vice versa.
- Kinetic energy will be maximum at mean position and zero at extreme position.
- Total mechanical energy is independent of time.
- Potential energy and kinetic energy peaks twice during every period. Element of springiness stores potential energy and element of inertia stores its kinetic energy.
- Graph of kinetic energy versus position of particle will be an inverted parabola.
- In absence of damping ; total mechanical energy of harmonic oscillator will remain constant.

  $E = k_{max} = U_{max} = \frac{1}{2}m\omega^2 A^2$
- Potential energy and kinetic energy is periodic with period $\frac{T}{2}$.
- The graphs for energy versus position are



6 OSCILLATIONS DUE TO A SPRING

(1) **Oscillations of a spring block system**



(Linear S.H.M.)

Force law, $F = -kx$

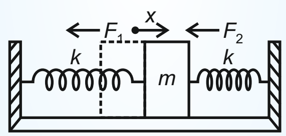
$F = -ma = -m\omega^2 x$

$\therefore \quad k = m\omega^2$ or $\omega = \sqrt{\frac{k}{m}}$

Where k spring constant of spring and m is mass of block executing SHM.

(2) **For two Identical Springs**

This is also linear harmonic oscillator



When displaced right, restoring forces towards left

$F_1 = -kx, F_2 = -kx, F = F_1 + F_2$

$F = -2kx$

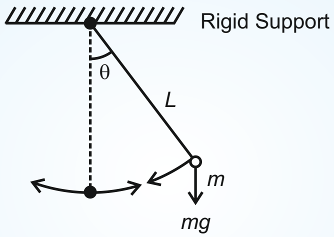
Since force acting on mass is proportional to displacement and directed towards mean position. It is SHM. The period of oscillation is

$T = 2\pi\sqrt{\frac{m}{2k}}$

7 SIMPLE PENDULUM

Simple Pendulum

- By attaching a small mass to an inextensible string, a simple pendulum can be made.
- The mass executes SHM for small displacements only.



$T = 2\pi\sqrt{\frac{I}{mgL}}$ Also here $I = mL^2$, about rigid support point.

$T = 2\pi\sqrt{\frac{L}{g}}$

- The time period of a simple pendulum depends on its length and acceleration due to gravity but is independent of its mass and amplitude.
- Time period of a clock pendulum which ticks every second is 2s and its length is approximately 1 metre.

9 FORCED OSCILLATIONS AND RESONANCE

- An external agency can maintain motion by resisting damping forces. These are called driven or forced oscillations. An external force which is periodic is applied to damped oscillator. Equation of oscillations of mass is $\frac{md^2y}{dt^2} + b\frac{dy}{dt} + ky = F_0 \cos\omega_d t$, and after natural oscillation, die out eqn. is

  $y = A\cos(\omega_d t + \phi)$ and A depends on ω_d and ω.
- If ω_d is close to ω then $A = \frac{F_0}{\omega_d b}$
- The phenomenon of increase of amplitude when driving frequency is close to natural frequency of the oscillators is called resonance.

8 DAMPED SIMPLE HARMONIC MOTION

- A viscous surroundings will apply force on simple pendulum or a spring pendulum and system will ultimately come to rest.
- The damping force depends on nature of surrounding medium. When damping is high, energy is quickly dissipated. This force is generally proportion to velocity of oscillator.

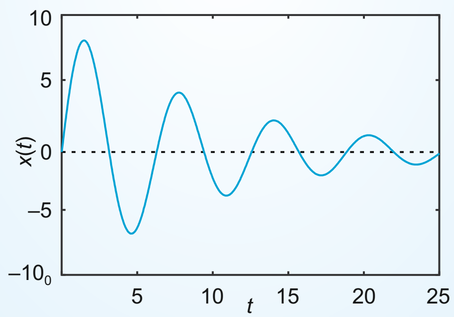
  $\vec{F}_d \propto \vec{v} \Rightarrow \vec{F} = -b\vec{v}$
- Net force $F = -ky - bv$ ($b \to$ damping factor)

  $ma = -ky - bv$

$\frac{md^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$ is damped equation, whose solution is given by

$y = Ae^{-\frac{bt}{2m}}\cos(\omega' t + \phi)$ for displacement of oscillator.

Where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$



- Small damping means $\frac{b}{\sqrt{km}} << 1$
- $\left[E = \frac{1}{2}kA^2e^{-bt/m}\right]$ energy eqn.

Chapter 15

Waves

WAVE

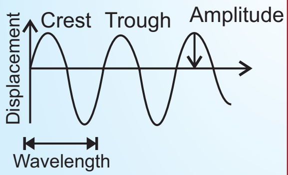
It is a disturbance produced, which transfer energy and momentum without transfer of matter.

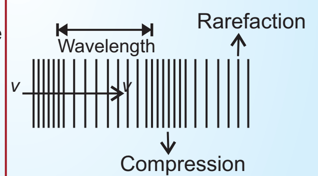
1 TYPES OF WAVES

- **Electromagnetic Wave** – wave propagates in the form of time varying electric and magnetic fields. It require no medium.
- **Matter waves** – wave associated with the particles having momentum.
- **Mechanical waves** – The waves which require a material medium for their propagation.

MECHANICAL WAVES

Transverse waves	Longitudinal waves
The individual particle of medium vibrate perpendicular to direction of propagation.	The individual particle of medium vibrate parallel to direction of propagation





- Trasverse waves are possible in solids like strings (under tension), due to shear modulus.
- Longitudinal waves, involve compressive stress, i.e. (Bulk modulus), so is possible in both solids and fluids
- Waves on the surface of water are of two kinds capillary waves and gravity waves

2 DISPLACEMENT RELATION IN A PROGRESSIVE WAVES

- $y(x, t) = a \sin(kx - \omega t + \phi)$

  a = amplitude of wave

  is linear combination of sine and cosine function

  $y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$

  Amplitude of resultant wave, $a = \sqrt{A^2 + B^2}$

  $\phi = \tan^{-1}\left(\frac{B}{A}\right)$

  Speed of wave $v = \frac{\omega}{k} = \nu\lambda$

  $k = \frac{2\pi}{\lambda}$ called angular wave number)

  $(kx - \omega t + \phi)$ = Phase of wave

- **Speed of a Transverse Wave on a Stretched String**

  $v = \sqrt{\frac{T}{\mu}}$

  Here

  T = tension in string (in newton)

  $\mu = \frac{m}{l}$ (mass per unit length of string)

Speed of a Longitudinal wave

- Speed of longitudinal wave in a solid bar $v = \sqrt{\frac{Y}{\rho}}$

  where Y = Young's modulus of material of bar

  ρ = Density of material of bar

- Speed of longitudinal wave in gases

  According to Newton, $v = \sqrt{\frac{P}{\rho}}$ (Isothermal)

  According to Laplace, $v = \sqrt{\frac{\gamma P}{\rho}}$ (Adiabatic)

3 PRINCIPLE OF SUPERPOSITION OF WAVES

- If $y_1(x, t)$ and $y_2(x, t)$ be the displacement due to two wave disturbances in the medium and the waves arrive in a region simultaneously and overlap, the net displacement $y(x, t)$ is given by

  $y(x, t) = y_1(x, t) + y_2(x, t)$

  Similarly, resultant waveform

  $$y = \sum_{i=1}^{n} f_i (x - vt)$$

  In the phenomenon of **interference** of two waves

  $y_1(x, t) = a \sin(kx - \omega t)$

  and $y_2(x, t) = a \sin(kx - \omega t - \phi)$

  The net displacement

  $$y(x, t) = 2a \cos\frac{\phi}{2} \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

  So, amplitude is a function of phase difference

  $$A(\phi) = 2a \cos\left(\frac{\phi}{2}\right)$$

  For ϕ, = 0, $A = 2a$ (Constructive interference)

  For constructive interference, path difference between two waves, $\Delta x = 0, \lambda, 2\lambda, \ldots. n\lambda$

  For $\phi = \pi$, $A = 0$ (Destructive interference)

  For destructive interference, path difference between two waves,

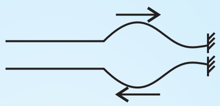
  $$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots. (2n - 1)\frac{\lambda}{2}$$

④ REFLECTION OF WAVES

- Rigid Boundary – At rigid boundary wave suffer a phase change of π.

$y_i = a\sin(\omega t - kx)$

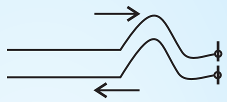
$y_r = -a\sin(\omega t + kx)$



- Open Boundary or Free boundary : At open boundary phase change is 0.

$y_i = a\sin(\omega t - kx)$

$y_r = a\sin(\omega t + kx)$



Standing Waves and Normal Modes

When two waves of same amplitude and of same frequency travel in opposite direction then resultant wave pattern from their superposition is called standing waves.

From open boundary.

$y_i(x, t) = a\sin(\omega t - kx)$,

$y_r(x, t) = a\sin(\omega t + kx)$

$y = y_i + y_r$

$y(x, t) = 2a\sin\omega t\cos kx$

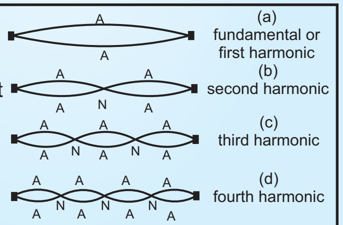
- The amplitude varies from point to point, but each element of string oscillate with same angular frequency (ω)
- **Nodes** – The point at which amplitude is zero or there is no motion called nodes. Distance between two consecutive nodes is $\lambda/2$.
- **Antinodes** – The points at which amplitude is maximum called antinodes. Distance between two consecutive antinodes is $\lambda/2$

Normal modes of stretched string Fixed At Both Ends

$L = \frac{n\lambda}{2}$, $n = 1, 2\ 3$

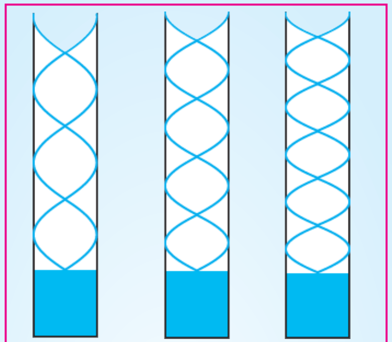
Frequencies of different modes

$\nu = \frac{nv}{2L}$, $n = 1, 2\ 3$--



⑤ NORMAL MODES OF ORGAN PIPES

Closed organ pipe



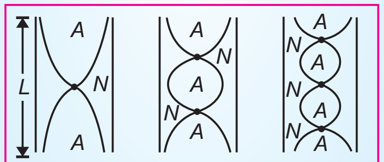
$L = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$: for $n = 0, 1, 2, 3,$.....

Possible wavelengths $\lambda = \dfrac{2L}{\left(n + \frac{1}{2}\right)}$

for $n = 0, 1, 2, 3$

Natural frequencies $\nu = \left(n + \frac{1}{2}\right)\frac{v}{2L}$, for n or $= 0, 1, 2, 3$

Open Organ Pipe



$L = n\frac{\lambda}{2}$, for $n = 1, 2, 3$...

Wavelength of stationary wave, $\lambda = \frac{2L}{n}$, for $n = 1, 2, 3$...

Frequencies of different modes, $\nu = \frac{nv}{2L}$, for $n = 1, 2, 3$...

- A compression is reflected as compression from the closed end of the organ pipe and as rarefaction from the open end.
- A rarefaction is reflected as rarefaction from the closed end of the organ pipe and as compression from the open end.

⑥ BEATS

- When two harmonic sound waves of nearly same frequencies travel in the same direction then the intensity of resultant wave produced from their superposition increase and decrease continuously at same point with time. It is called beat formation.
- Two waves of angular frequencies ω_1 and ω_2 superimpose at , $x = 0$ at time t

$s_1 = a_1\cos\omega_1 t$, $s_2 = a_2\cos\omega_2 t$

from superposition, $s = s_1 + s_2$

$s = 2a\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t.\cos\left(\frac{\omega_1 + \omega_2}{2}\right)t$

$\omega_b = \frac{(\omega_1 - \omega_2)}{2}$ and $\omega_a = \frac{(\omega_1 + \omega_2)}{2}$

Beat frequency, $\nu_{beat} = |\nu_1 - \nu_2|$

- We hear a waxing and warning of sound with frequency equal to difference between the frequencies of superposing waves.

⑦ DOPPLER EFFECT

Generally, if there is relative motion between a source(s) and observer then observed frequency will be other than real frequency. This apparent change in frequency is called Doppler effect.

- Both source and observer moving

$\nu = \nu_0\left(\frac{v + v_0}{v + v_s}\right)$

here v is the speed of sound through the medium, ν_0 is the velocity of observer relative to the medium, and v_s is the source velocity relative to the medium. In using this formula, velocities in the direction O to S should be treated as positive and those opposite to it should be taken to be negative.

- When source and observer stationary and wind is blowing towards stationary observer with speed v_w, apparent wavelength

$\lambda_a = \frac{(v_0 - v_w)}{\nu}$

- When source is moving towards the stationary observer with medium at rest, apparent wavelength

$\lambda_a = \frac{(v - v_s)}{\nu}$

Electric Charges and Fields

1 Chapter

1 ELECTRIC CHARGE

Positive and negative charges were named by Benjamin Franklin.

Charging can be done by

1. Friction 2. Induction 3. Conduction

Charging by friction

When glass rod is rubbed with silk, the rod acquires one type of charge and silk acquires other type of charge.

2 PROPERTIES OF CHARGES

- Two types of charges exist.
- Like charges repel unlike attract.
- A body is charged by loss or gain of electrons.
- In an isolated systems, total charge remains conserved.
- Charge exists in discrete nature. $q = \pm n \times e$
- Moving charge has magnetic effects along with electric effects.
- Accelerating charges emit radiations.
- Gold leaf electroscope detects charge on a body.
- Charge is scalar and additive in nature.

3 CONDUCTORS AND INSULATORS

- Some substances which readily allow passage of electricity through them are called conductors
- Metals, human body and earth are conductors.
- Materials which opposes flow of charge through them are insulators.
- Glass, porcelain, plastic, nylon, wood etc are insulators.

Earthing

A process of sharing charges with earth is called grounding or earthing

4 CHARGES INTERACTION

Coulomb's law is quantitative statement about force between two point charges.

- Force varies inversely as square of distance between the charges and directly proportional to product of magnitude to two charges and acts along the line joining two charges
- Two charges q_1 and q_2 separated by distance r in vacuum, the magnitude of force (F) between them

$$F = K\frac{|q_1q_2|}{r^2}$$

K depends on system of units and medium. In SI unit in vacuum $K = 9 \times 10^9$. Unit of charge is coulomb(C)

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(q_1q_2)}{r^2}, \ \varepsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$$

5 FORCE BETWEEN MULTIPLE CHARGES

Force on any charge due to number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time, the individual forces are unaffected by presence of other charges. This is termed as superposition principle of electrostatics. Vector sum of forces is obtained by parallelogram law of addition of vectors. Force on first charge due to other is given as $\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$

6 ELECTRIC FIELD OF GHARGES

- A charge placed at a point produces an electric field everywhere in the surrounding. When another charge is brought in field, field there acts on it and produces a force. Faraday introduced field concept.
- Electric field intensity produced by a charge Q at a point distance r is given by

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

- SI unit of electric field is N/C. Field intensity at a point due to charge Q in space is defined as the force that a unit positive charge would experience if placed at that point.
- $\vec{E} = \lim_{q \to 0}\left(\frac{\vec{F}}{q}\right)$
- Field vary from point to point and is a vector quantity. Field can transport energy.

Field Due to System of Charges

Electric field at a point P in space due to system of charges is defined as force experienced by a unit test charge placed at that point

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{ip}^2}\hat{r}_{ip}$$

7 ELECTRIC DIPOLE

- An electric dipole is an arrangement of pair of equal and opposite point charges separated by a distance.
- Direction from $-q$ to $+q$ is direction of dipole moment.

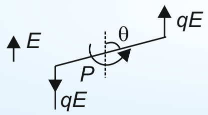
Electric fields due to dipole

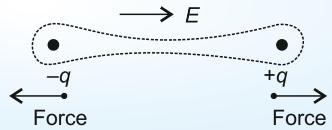
On Axis of Dipole	On Equatorial Plane
$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}r}{(r^3 - a^2)^2} \approx \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$ (if $r >> a$)	$\vec{E} = \frac{-1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2 + a^2)^{3/2}} \approx \frac{-1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^3)}$ (if $r >> a$)

- Polar molecule : H_2O have permanent electric dipole moment even in absence of electric field.
- Non polar molecule : CH_4, CO_2. The Dipole moment is zero.
- A dipole in external uniform electric field experience torque but no net force.

$\vec{\tau} = \vec{p} \times \vec{E}$

$|\tau| = PE \sin\theta$

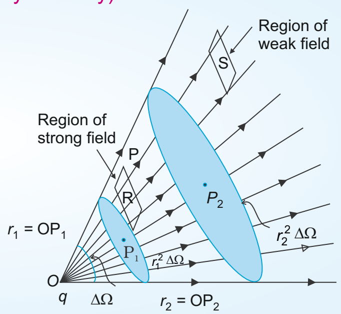




8 ELECTRIC FIELD LINES

(Lines of force by Faraday)

Field lines carry information about direction of electric field at different points in space. Relative density of field lines at different points indicates relative strength of electric fields at these points. Faraday introduced non mathematical way of visualizing electric field around charge configuration.



9 ELECTRIC FIELD LINES PROPERTIES

- It is a way of pictorially mapping the electric field around a configuration of charges.
- A line is a curve drawn in such a way that tangent to it at each point is in the direction of net field at that point.
- A field line is a space curve (A curve in three dimension)
- Field lines of a single positive charge are radially outward.
- Field lines start from positive charges and end at negative charges.
- From a single charge, line can start from or end at infinity.
- In a charge free region, electric field lines can be taken to be continuous curves without any breaks.
- Two field lines can never cross each other.
- Electrostatic field lines don't form any closed loop. This follows from their conservative nature.

10 ELECTRIC FLUX

- Similar to fluid flow an analogous quantity exists in electrostatic called electric flux. There is no flow of observable quantity.
- The number of (field) lines crossing a unit area placed normal to field at a point is measure of strength of electric field at that point.
- Number of field lines crossing ΔS area is proportional to $E\Delta S\cos\theta$. This is called electric flux through area element ΔS.

$\Delta\phi = \vec{E}\cdot\Delta\vec{S} = E\Delta S\cos\theta$

θ = angle between $\vec{E}$ and outward drawn normal to area element $\Delta\vec{S}$.

- Units : $NC^{-1}m^2$
- Total flux $\phi \approx \Sigma\vec{E}\cdot\Delta\vec{S}$

Approximate sign is because electric field is taken uniform over area element.

If $\Delta S \to 0$ then, $\phi = \oint \vec{E}\cdot\vec{ds}$

11 GAUSS'S LAW

- Total electric flux through closed surface $s = \frac{q}{\varepsilon_0}$, where q = Total charge enclosed by s
- Total flux is zero if closed surface encloses no charge.
- Gauss law is true for any surface, no matter what its shape or size is.
- q is total charge enclosed by surface, located anywhere inside.
- Gaussian surface should not pass through discrete charges.
- Any violation of Gauss's law will indicate departure of inverse square law.

Application of Gauss's law

- Electric field due to infinitely long wire

  $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, at distance r from linearly charged rod.

- Field of Uniformly Charged Shell

  $E = \frac{q}{4\pi\varepsilon_0 r^2}\ (r \geq R)$

  Directed out for $q > 0$, directed inwards for $q < 0$

  $E = 0\ \ (r < R)$ field is zero inside shell.

- Electric field due to infinite plane sheet

  $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$ is independent of distance from sheet.

Electrostatic Potential and Capacitance

2
Chapter

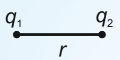
1 ELECTROSTATIC POTENTIAL ENERGY

- Work done by external force in moving a charge against electrostatic repulsive force gets stored in it as potential energy.
- Electric potential energy difference between two points is work required to be done by an external force in slowly moving charge from one point to another against electric field of any charge configuration.
- Potential energy of a charge at a point in electric field due to any charge configuration, is the work done by external force in slowly bringing the charge from infinity to that point.

$$U = \int_{\infty}^{r} \vec{F}_{ext} \cdot \vec{dr} \quad = -\int_{\infty}^{r} \vec{F}_{E} \cdot \vec{dr}$$

2 POTENTIAL ENERGY OF A SYSTEM OF CHARGES

- For assembly of two charges

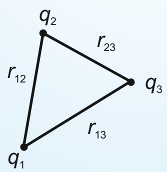


for q_1 and q_2 at separation r

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$ (Depends on charge nature)

- For assembly of three charges

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$



3 ELECTRIC POTENTIAL

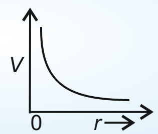
Work done by an external force in bringing a unit positive charge from infinity to that point without acceleration is equal to electrostatic potential at that point.

Its SI unit is volt.

4 ELECTROSTATIC POTENTIAL DUE TO A POINT CHARGE

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

For Q > 0, V > 0
For Q < 0, V < 0



5 POTENTIAL DUE TO A SYSTEM OF CHARGES (SUPERPOSITION LAW)

- Potential at a point due to total charge configuration is the algebraic sum of the potentials due to individual charges

$$V = V_1 + V_2 + V_3 + ... = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$$

6 POTENTIAL DUE TO AN ELECTRIC DIPOLE

- $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

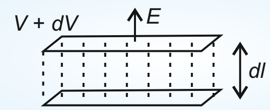
  ($r >>$ dipole length at general point)

- Potential on axis of dipole

$$V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \quad \begin{bmatrix} +\text{For } \theta = 0 \\ -\text{For } \theta = \pi \end{bmatrix}$$

- Potential in the equatorial plane of dipole is zero

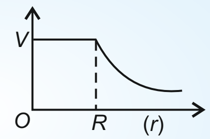
7 RELATION BETWEEN FIELD AND POTENTIAL



$E = -dV/dl$

8 POTENTIAL DUE TO UNIFORMLY CHARGED SPHERICAL CONDUCTING SHELL

- $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ $(r \geq R)$



  q is charge on shell and R is its radius.

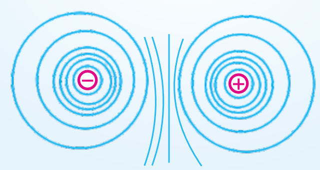
- Potential is constant inside shell and is equal to potential at surface.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

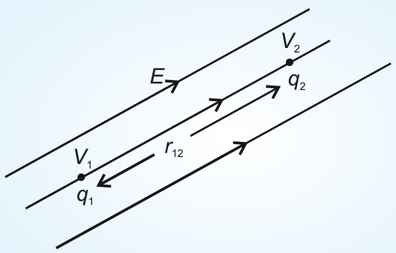
9 EQUIPOTENTIAL SURFACES

- It is a surface with a constant value of potential at all points on its surface.
- Equipotential surfaces of a single point charge are concentric spherical shells centered at the charge.
- For any charge configuration, an equipotential surface is normal to electric field at that point
- No two equipotential surfaces cut each other.

  For dipole : Equipotential surfaces :



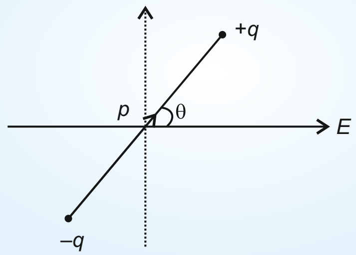
(10) POTENTIAL ENERGY IN EXTERNAL FIELD



$$U = q_1V_1 + q_2V_2 + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$$

(11) POTENTIAL ENERGY OF A DIPOLE

$$U(\theta) = -pE\cos\theta = -\vec{p}.\vec{E}$$



(12) ELECTROSTATICS OF A CONDUCTOR

- Inside conductor, electrostatic field is zero, either is neutral or charged.
- Electrostatic potential is constant throughout volume of the conductor & same value as on surface.
- If a cavity is created inside conductor and a charge is kept outside cavity. Any electric field outside conductor does not enter into the cavity. So cavity of conductor remains shielded.
- No work done is done in moving a charge on a conducting surface.
- When a conductor placed in external electric field, field lines are always normal to conducting surface.

(13) DIELECTRICS

- Dielectrics are non conducting substances having no charge carriers.
- Polar dielectric : Polar dielectric has permanent dipole moment. Ex. HCl, H_2O.
- Non polar dielectric : Non polar dielectric has no dipole moment. Ex. O_2, H_2.
- A dielectric with polar/non polar molecules develops a net dipole moment in an external electric field. The dielectric is polarized. Dipole moment developed per unit volume called polarization P.

  $P = \varepsilon_0\chi_e E$, χ_e = electric susceptibility of dielectric medium.

  $\chi_e = (K-1)$

(14) DIELECTRIC STRENGTH

- Maximum value of electric field that a dielectric medium can withstand without breakdown (of its insulating property) is called its dielectric strength .
- For air dielectric strength is

  $E = 3 \times 10^6$ V/m
- For any capacitor, the electric field do not exceed the break down limits. There is limit to charge amount that can be stored on a given capacitor without significance leakage.

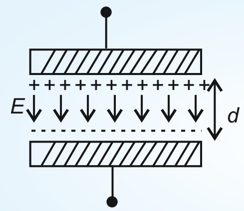
(15) CAPACITANCE OF CAPACITORS

$$C = \frac{Q}{V}$$

C is independent of Q and V but depends on shape, size and separation of system of two conductors & also on dielectric, separating two conductors. Every capacitor has limited electric capacity.

SI unit : F (farad)

(16) PARALLEL PLATE CAPACITOR



$$C = \frac{\varepsilon_0 A}{d}$$

Plate area : A (For each)

Plate separation : d

Dielectric inserted occupying full intervened region

$$C = \frac{K\varepsilon_0 A}{d}$$

K = dielectric constant of the substance

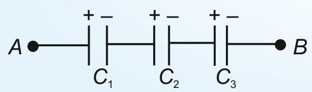
(17) COMBINATION OF CAPACITORS

Series Combination of Capacitors

Charges on plates ± Q are same on each capacitor.

$$V = V_1 + V_2 + V_3$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



- Equivalent capacity decreases.

Parallel Combination of Capacitors

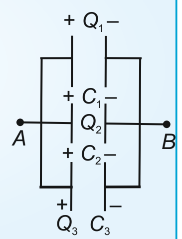
Same potential difference is applied across each capacitor.

Plate charges not necessarily same.

Equivalent capacity

$C = C_1 + C_2 + C_3$

$Q = Q_1 + Q_2 + Q_3$



- Equivalent capacity increases in parallel

(18) ENERGY STORED IN A CAPACITOR

- $U = \frac{QV}{2} = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$
- $U/V = u = \frac{1}{2}\varepsilon_0 E^2$

  (Enegry density)

(19) CHARGE SHARING IN CAPACITORS

- When two capacitors of different potential are joined with positive plates together and negative together, common potential is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

- Final energy is less than initial and is lost as heat and electromagnetic radiation

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

Current Electricity

3
Chapter

1 ELECTRIC CURRENT

Current through a given area is net charge passing per unit time through the area.

- Current may not always be steady. We define current in general

$$I = \lim_{\Delta t \to 0}\left(\frac{\Delta Q}{\Delta T}\right)$$

- Its SI unit is ampere (A)
- A cell can maintain a steady current

2 DRIFT VELOCITY

The charge carriers like electrons move with an average velocity which is independent of time, this is phenomenon of drift, and is called drift velocity.

$$\vec{v}_d = \frac{-e\vec{E}}{m}\tau$$

τ = relaxation time.

Although collision of electrons don't occur at regular intervals but average time between successive collision is taken as relaxation time.

4 OHM's LAW

The current flowing through a conductor is proportional to potential difference across it, provided temperature is constant.

$$V \propto I \text{ or } V = RI$$

R is the resistance of substance. SI unit of is ohm ($1\,\Omega = 1\,\text{V}\,\text{A}^{-1}$)

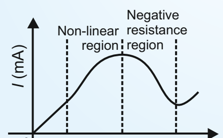
Equivalent form: $\vec{J} = \sigma\vec{E}$ ($\vec{J}$: Current density vector)

Factors affecting *R*: $R = \frac{\rho l}{A}$

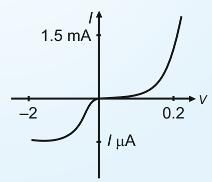
1. Material of conductor
2. Area of cross-section of conductor
3. Length of conductor,

Limitation of ohm's law

- The relation of V and I is not unique in GaAs.
- V ceases to be proportional to *I*. Material becomes non-ohmic material.
- For a diode, relation of *V* and *I* depends on sign of *V*. This material is used in electronic devices.



V (GaAs)
(not obeying ohm's law)



5 RESISTIVITY AND ITS TEMPERATURE DEPENDENCE

- Materials are classified as conductors, semiconductors and insulators according to their resistivity value.
- Metals have resistivity range $10^{-8}\,\Omega\,\text{m}$ to $10^{-6}\,\Omega$ m.
- Insulators have resistivity range from 10^{5} to $10^{18}\,\Omega$ m.
- For metallic conductor over a limited range, resistivity is approximately given by

$\rho_T = \rho_0[1 + \alpha(T - T_0)]$

ρ_T = resistivity at temp. T

ρ_0 = resistivity at temp. T_0

α = temperature coefficient of resistivity

6 TYPES AND COLOUR CODING OF RESISTORS

(a) Wire Bound Resistors

- Made of materials which are relatively insensitive to temperature.
- Winding of wires are of alloys viz., manganin, constantan, nichrome etc.
- Range : fraction of an ohm to few hundred ohms.

(b) Carbon Resistors

- Compact, inexpensive and have higher range.
- Colour coding of carbon resistors

Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

3 CURRENT DENSITY AND MOBILITY

Current through unit cross-sectional area is called current density.

- It is denoted by J and is a vector.
- SI unit is $\text{A}\,\text{m}^{-2}$
- $\vec{J} = \sigma\vec{E} = \left(\frac{ne^2}{m}\tau\right)\vec{E}$

  σ = conductivity

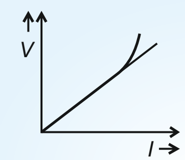
  E = electric field inside conductor

  The relation is Ohm's law in microscopic form.

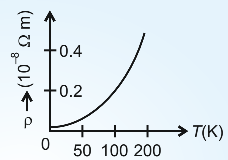
- Conductivity is due to mobile carriers.
- In metals, charge carriers are electrons.
- In ionised gas, they are electrons and positive charged ions.
- In electrolytes they are positive and negative ions.
- Mobility is magnitude of drift velocity per unit electric field.

$$m = \frac{|v_d|}{E} = \frac{e\tau}{m}$$

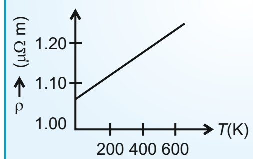
- SI units are $\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$



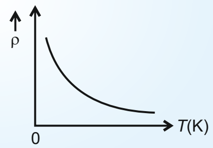
(For good conductor)



(Copper)



(Nichrome)



Semi-conductors

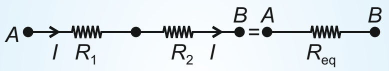
7 CELL AND ITS EMF

- It is a simple device which can maintain a steady current in electric circuit.
- EMF of cell is potential difference between positive and negative electrode when no current is flowing through the cell.
- $V = \varepsilon - ir$ (discharging)
  $V = \varepsilon + ir$ (charging)
- r is called internal resistance. The actual value of r vary from cell to cell.
- Internal resistance of dry cell is higher than electrolytic cell.

8 COMBINATION OF RESISTORS

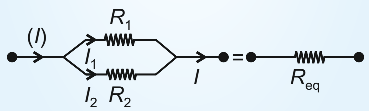
The resistors are sometimes joined together and there are simple rules for calculation of equivalent resistance of such combination.

1. **Series combination:** If only one of their end point is joined.



$R_{eq} = R_1 + R_2$

2. **Parallel combination:** If one end of all the resistors are joined together and similarly other ends joined together. (The potential drop across resistors is same).



$I = I_1 + I_2$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

9 Electrical Energy and Power

Under a potential difference in a conductor charges are moving. These charges suffer collisions with ions and atoms during transit. Energy shared by ions and atoms heats up the conductor. Amount of energy dissipated as heat per unit time is called power loss.

$$P = I^2R = V^2/R = IV$$

R is resistance when current I is flowing through it.

This energy is supplied by source in circuit.

- For long distance transmission, power loss is minimised by transmitting it at high voltage.

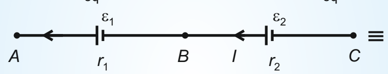
10 KIRCHHOFF'S RULES

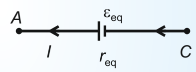
- For complicated electric circuits to determine all the currents and potential differences, Kirchhoff formulated two laws:
  - **Junction rule:** At any junction, sum of currents entering the junction is equal to sum of currents leaving the junction.
  - **Mesh or loop rule:** The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is equal to zero.

Note: Binding or reorientation of wire does not change the validity of junction law.

11 COMBINATION OF CELLS

- Cell can be grouped in series or parallel depending upon current requirements.
- In series: Two cells of emf ε_1 and ε_2 with internal resistances r_1 and r_2 the combination can be considered as one cell of emf ε_{eq} and internal resistance r_{eq}

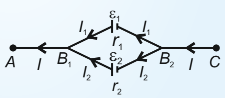




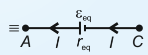
$(\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2)$ and $(r_{eq} = r_1 + r_2)$

- In parallel combination of two cells

$$\varepsilon_{eq} = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}\right)$$



$$r_{eq} = \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

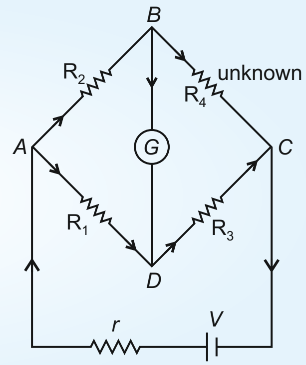


12 WHEATSTONE BRIDGE

- Wheatstone bridge in its balanced condition provide a practical method for determination of internal resistance.
- If R_1 and R_2 are two resistances in first and second arm and R_3 in third arm. R_3 is kept on changing till galvanometer shows no deflection. The bridge is then balanced and from balance condition R_4 is known.

$$R_4 = R_3 \times \frac{R_2}{R_1}$$

- The value of one resistance is determined knowing other three resistors.

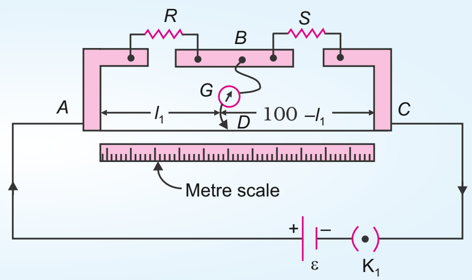


13 METER-BRIDGE

- It is based on wheatstone bridge.
- With same principle as of Wheatstone bridge it is used to calculate unknown resistance, R, under balance condition.

$$R = S \times \frac{l_1}{100 - l_1}$$

- Percentage error in R is minimised by adjusting balance point near the middle of bridge.



14 POTENTIOMETER

This is a versatile electric instrument used to compare emf(s) and to determine internal resistance of a cell.

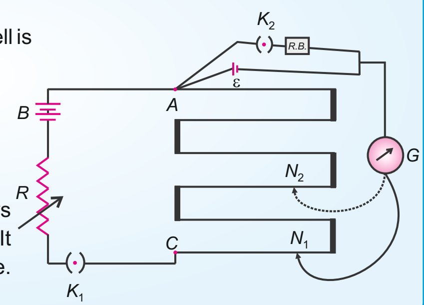
- The method involves condition of no current flow. In this way it can compare emfs of two cells.
- The potentiometer wire has uniform cross-section and homogeneous material so potential drop per unit length of potentiometer wire is constant.
- The formula for internal resistance calculation of cell is

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

l_1 = balancing length without shorting cell

l_2 = balancing length with cell by parallel resistance R

- The potentiometer has the advantage that it draws no current from voltage source being measured. It is not affected by internal resistance of the source. Thus it has high accuracy.

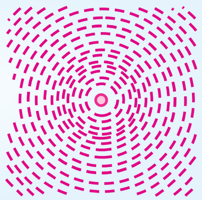


Moving Charges and Magnetism

Chapter 4

1 MAGNETIC FIELD

- It is space around a current carrying conductor in which its magnetic effects can be felt.
- Oersted concluded that moving charges or currents produced a magnetic field in the surrounding space.



2 LORENTZ FORCE

Mechanical force experienced by a moving charge through electric and magnetic field

$F = q\left[\vec{E} + (\vec{v} \times \vec{B})\right] = \vec{F}_{electric} + \vec{F}_{magnetic}$

- Magnetic force depends on magnitude of charge, its nature and its velocity.
- When charge is at rest, it does not experience any magnetic force.
- When charge is moving parallel to magnetic field, it does not experience any mechanical force.

3 VELOCITY SELECTOR

When electric field and magnetic fields are crossed and velocity of particle is perpendicular to both fieldsthen particles with speed $v = \frac{E}{B}$ pass undeflected. This principle is employed in mass spectrometer.

4 MOTION IN MAGNETIC FIELD

In uniform magnetic field charge particle can have three types of path.

1. Straight line: when $\vec{B} \,||\, \vec{v}$
2. Circular path: $\vec{v} \perp \vec{B}$

   Perpendicular force acts as a centripetal force and produces a circular motion perpendicular to magnetic field.

   Radius of circle $r = \frac{mv}{qB}$ and $T = \frac{2\pi m}{qB}$
3. Helical path: velocity $\vec{v}$ and $\vec{B}$ are inclined at angle

   $\theta \neq 0,\ \theta \neq 90°,\ \theta \neq 180°$

   Velocity component along magnetic field remains unchanged, due to other component motion is circular. The combined path is helical motion.

   $r = \frac{mv_{\perp}}{qB}, \omega = \frac{qB}{m},\ p = \frac{2\pi m v_{||}}{qB}$

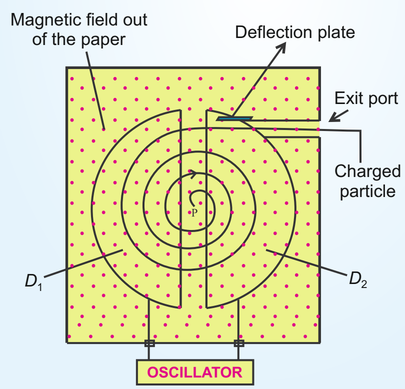
5 CYCLOTRON

A machine to accelerate charged particles or ions to high energies cyclotron; uses both electric and magnetic field in combination to increase kinetic energy of charge particles

- Frequency of revolution of charge particle is independent of its energy.
- $f = \frac{qB}{2\pi m}$. The frequency is called cyclotron frequency.

The frequency of electric field is in resonance with cyclotron frequency. Final KE of ion

$E_K = \frac{q^2B^2R^2}{2m}$, R = radius of Dee



6 BIOT-SAVART'S LAW

- According to this law, the magnetic field at a point due to a current element of length dl carrying current I at distance r from element is

  $|d\vec{B}| = \frac{\mu I dl \sin\theta}{4\pi r^2}$

  θ is angle between $\vec{dl}$ and $\vec{r}$

7 MAGNETIC FIELD ON AXIS OF CIRCULAR COIL

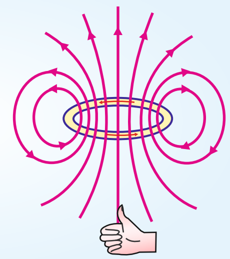
$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$

where R = radius of coil

x = distance along axis from centre of coil plane

- At the centre of loop, $x = 0$

  $B = \frac{\mu_0 I}{2R}$



Field lines form closed loop around circular wire

8 AMPERE'S CIRCUITAL LAW

Law states $\oint_c \vec{B} \cdot \vec{dl} = \mu_0 I$, where I refers to current passing the loop through open surface S. The sign of current is determined from right hand rule.

- If B is directed along tangent to amperian loop of perimeter L and field is constant in magnitude

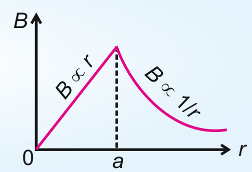
  $BL = \mu_0 I_e$

  I_e = net current enclosed by closed loop.

9 MAGNETIC FIELD DUE TO SOLID CONDUCTOR

A long straight wire with circular cross-section of radius a

- Magnetic field in region $r < a$, $B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r$
- Magnetic field in region $(r \geq a)$, $B = \frac{\mu_0 I}{2\pi r}$



10 MAGNETIC FIELD DUE TO A LINE CURRENT

- Magnetic field at distance R from straight long infinite wire carrying a current I.

  $B = \frac{\mu_0 I}{2\pi R}$, field lines are circles concentric with wire.

11 DIRECTION OF MAGNETIC FIELD

The rule is called right hand rule:

Grasp the wire in your right hand with your extended thumb pointing in the direction of the current, your fingers will curl around in the direction of magnetic field.

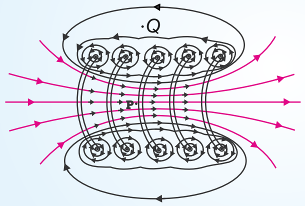
12 LONG SOLENOID

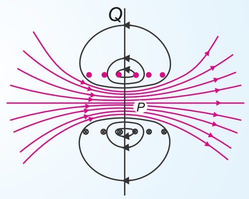
Magnetic field inside long solenoid (B)

When solenoid carries current I is

$B = \mu_0 nI$

n = number of turns per unit length





FOR A TOROID

$B = \frac{\mu_0 NI}{2\pi r}$

N = total number of turns and r = average radius

13 MECHANICAL FORCE ON A CURRENT CARRYING CONDUCTOR

A current carrying conductor of straight length L carrying current I experience force

$\vec{F} = I(\vec{l} \times \vec{B})$

But if wire is of arbitrary shape

$\vec{F} = \sum i(\vec{dl} \times B)$

Summation can be converted into integration in most cases

14 FORCE BETWEEN CURRENT CARRYING WIRES

- Two current carrying conductors placed near each other experience magnetic forces.

  When conductors are parallel

  $F = \frac{\mu_0 I_1 I_2}{2\pi d} L$

- Force on one conductor of length L due to current in other at separation d. Parallel currents attract and antiparallel currents repel. The results are in accordance with Newton's third law.

15 TORQUE ON CURRENT LOOPS

- Torque on magnetic dipole

  $\tau = mB\sin\theta$

  $\tau = NIA\sin\theta$

- Any planner current loop is equivalent to magnetic dipole of dipole moment

  $m = IA$

16 MAGNETIC MOMENT OF REVOLVING CHARGED PARTICLES

- Magnetic moment associated with revolving electron with speed v in a radius of circle r is

  $\mu = -\frac{evr}{2}$

  and $\vec{\mu} = \frac{-e}{2m_e}\vec{J}$

Where $\vec{J}$ is angular momentum of the electron

- For electron, angular momentum is opposite in direction to magnetic moment.
- In general for any charge q angular momentum and magnetic moment are in same direction.

  $\frac{\mu}{J} = \frac{e}{2m}$

This is called Gyromagnetic ratio and is constant.

Minimum value of magnetic moment is called Bohr magneton

$\mu_m = 9.27 \times 10^{-24}\ \text{Am}^2$

17 MOVING COIL GALVANOMETER

- Torque due to radial magnetic field on loop of area A with N number of turns carrying current I is

  $\tau = NIAB$

- deflection on scale

  $\phi = \left(\frac{NAB}{K}\right) I$

- Quantity in bracket is constant for galvanometer. This makes linear scale
- Current sensitivity of galvanometer

  $S_i = \frac{\phi}{I} = \frac{NAB}{K}$

- Current sensitivity can be easily increased by changing N

18 GALVANOMETER CONVERSION AMMETER

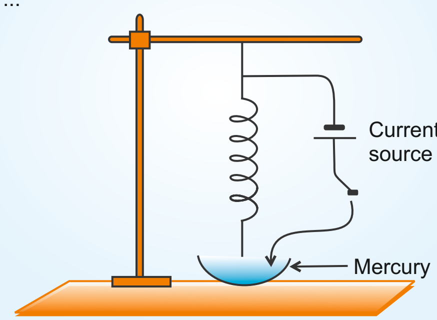
- Modification of galvanometer by connecting a low resistance in parallel.

VOLTMETER

- To measure voltage across any section of circuit. It is connected in parallel. When a large resistance is in series with galvanometer, it becomes a voltmeter.

19 ROGET'S SPIRAL

- When current passes through spring the effect is length of parallel current produces attraction, decreasing spring length, oscillations starts and continue with tick - tick - tick ...

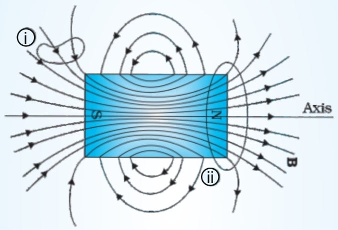


Magnetism and Matter

5
Chapter

1 BAR MAGNET

- It is a magnet in form of a bar
- When freely suspended, it points in N - S direction
- Like poles repel each other, unlike poles attract each other
- Magnetic monopoles do not exist
- Magnetic field lines of magnet form continous closed loops
- The tangent at a given point represents the direction of net magnetic field $\vec{B}$ at that point
- Magnetic field lines do not intersect each other.



- When magnet cut transverse to length or along its length $M' = M/2$
- Bar magnet as an equivalent solenoid
  $B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$
- Magnetic moment of solenoid
  $m = nI(2l) \times (\pi a^2)$
- Bar magnetic in uniform magnetic field
  $\vec{\tau} = \vec{m} \times \vec{B}$
  $U_m = -\vec{m} \cdot \vec{B} = -mB\cos\theta$
- $\theta = 0°$ (Most unstable position)
- $\theta = 180°$ (Most unstable position)
- Time period of oscillation of a magnet when freely suspended

$$T = 2\pi\sqrt{\frac{I}{mB}} \text{ and } B = \frac{4\pi^2}{mT^2}$$

2 THE ELECTROSTATIC ANALOGUE

Magnetism	Electrostatics
$\vec{B}$	$\vec{E}$
$\vec{m}$	$\vec{P}$
μ_0	$\frac{1}{\varepsilon_0}$
$\vec{\tau} = \vec{m} \times \vec{B}$	$\vec{\tau} = \vec{P} \times \vec{E}$
$U = -\vec{m} \cdot \vec{B}$	$U = -\vec{P} \cdot \vec{E}$
$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{r^3}$	$E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{P}}{r^3}$
$\vec{B}_{eq} = \frac{-\mu_0}{4\pi}\frac{\vec{m}}{r^3}$	$\vec{E}_{eq} = \frac{-1}{4\pi\varepsilon_0}\frac{\vec{P}}{r^3}$

3 GAUSS'S LAW FOR MAGNETISM

- $\oint \vec{B} \cdot d\vec{A} = 0$
- Isolated magnetic poles do not exist.
- The net magnetic flux is zero for any closed surface.

5 MAGNETISM AND MAGNETIC INTENSITY

- $\vec{M} = \frac{\vec{m}_{net}}{V} = \frac{\text{Net magnetic moment}}{\text{Volume}}$
- Net field in the interior of a solenoid
  $\vec{B} = \vec{B}_0 + \vec{B}_m$
  $\vec{B}_0$: Field in free space
  $\vec{B}_m$: Field contributed by material core

$\vec{B}_0 = \mu_0\vec{H}$ $\quad$ $\vec{B}_m = \mu_0\vec{M}$

$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)$ $\quad$ $\vec{M} = \chi\vec{H}$

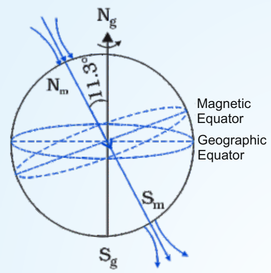
$\vec{B} = \mu_0(1+\chi)\vec{H}$ $\quad$ $\mu_r = 1 + \chi$

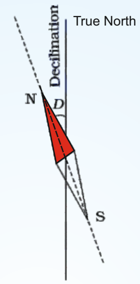
$\vec{B} = \mu_0\mu_r\vec{H}$ $\quad$ $\mu = \mu_0\mu_r = \mu_0(1+\chi)$

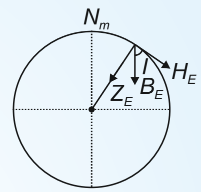
$\vec{B} = \mu\vec{H}$

4 EARTH'S MAGNETISM

- Magnetic field of earth is now thought to arise due to electrical current produced by convective motion of metallic fluids in outer core of the earth. This is known as dynamo effect .
- Magnetic poles inside earth change position with times.

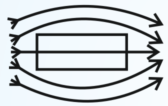
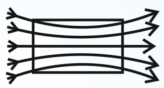
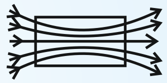
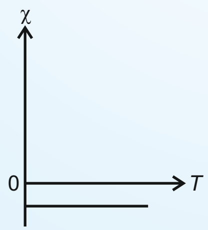
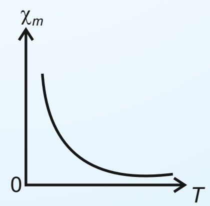
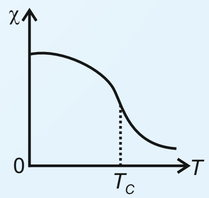






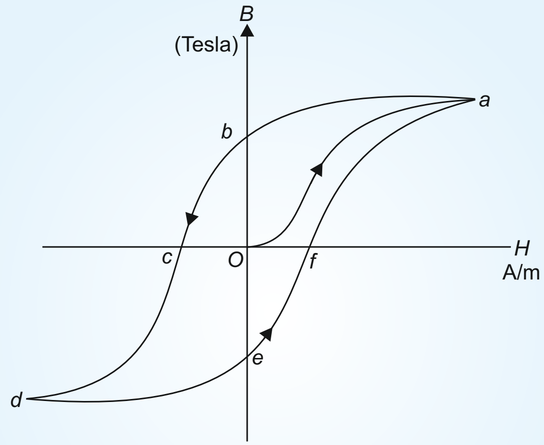
- Earth's magnetic field varies from point to point on earth surface, its value being of order of 10^{-5} T.
- Magnetic field lines of earth resemble that of a hypothetical magnetic dipole located at the centre of earth.
- The vertical plane which passes through the imaginary line joining the magnetic north and the south poles is called magnetic meridian.
- The vertical plane containing longitudinal circle and axis of rotation of earth is called geographic meridian.
- **Inclination or magnetic dip(I)**
  Angle made by the net magnetic field of earth with the horizontal
  $I_{equator} = 0$
  $I_{pole} = 90°$
- **Horizontal component (H_E)**
  Horizontal Component of net magnetic field of earth
- **Magnetic declination(D)**
  Angle between magnetic meridian and geographic meridian
- Declination in India is small. It is 0°41′ E at Delhi.
- Declination is greater at higher latitudes and smaller near equator.
- $B_E\sin I = Z_E$, $B_E\cos I = H_E$, $\tan I = Z_E/H_E$

6 MAGNETIC PROPERTIES OF MATERIALS

Properties	Diamagnetic	Paramagnetic	Ferromagnetic
χ	$-1 \le \chi \le 0$	$0 < \chi < k$ (k is a small positive number)	$\chi >> 1$
μ_r	$0 \le \mu_r < 1$	$1 < \mu_r < 1 + k$	$\mu_r >> 1$
μ	$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$
Magnetisation	Weak magnetisation is opposite direction	Weak magnetisation in same direction	Strong magnetisation in same direction
Movement in non-uniform magnetic field	(Weak tendency) from strong to weak magnetic field	(Weak tendency) from weak to strong magnetic field	(Strong tendency) from weak to strong magnetic field
Magnet	Weak Repulsion	Weak Attraction	Strong Attraction
E.g.	Bi, Au, Pb, Si, H_2O, NaCl N_2 (STP), Ag, superconductor	Al, Na, O_2 (STP), Pt, W	Fe, Co, Ni, Fe_2O_3 and Gd
Mag. Field lines			
Susceptibility	Independent of temperature $\chi_m \propto T^0$ 	Inversely proportional to temperature $\chi_m = \frac{C\mu_0}{T}$ Where C is curie constant 	Curie temperature: The temperature at which ferromagnetic substance becomes paramagnetic Disappearance of magnetisation with temperature is gradual. Beyond curie temperature Susceptibility obeys Curie-Weiss law $\chi_m = \frac{C}{T - T_C} \; (T > T_C)$ T_C for iron is 1043 K T_C for nickel is 631 K 

- Domain formation is special characteristic of ferromagnetic material. Typical domain size is 1 mm and domain contains about 10^{11} atoms.

7 HYSTERESIS LOOP



- **Retentivity:** The value of B at $H = 0$
- **Coercivity:** The value of H for which net magnetic field becomes zero.
- Energy is lost in the form of heat during complete cycle of magnetisation and de-magnetisation.
- Hysteresis curve of soft iron is tall and narrow.
- Hysteresis curve of steel is broad

8 PERMANENT MAGNETS AND ELECTROMAGNETS

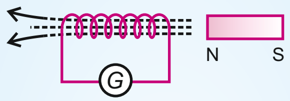
Permanent Magnets	Electromagnets
Material should have	Material should have
High Retentivity	Low Retentivity
High Coercivity	High permeability
High permeability	Low Coercivity
Material used for making these magnets are Alnico, Cobalt, Steel, Ticonal	Material used for making these magnets is soft Iron
These materials retain their ferromagnetic property for long period of time.	Electromagnets are used in electric bells, loud speakers and telephone diaphragms.

Electromagnetic Induction

6 Chapter

1 FARADAY'S EXPERIMENTS

(a) First experiment : Relative motion between a bar magnet and wire loop produces a small amount of current.



(b) Second experiment : If one coil is connected to a battery and another coil is moved towards or away from it, electric current is produced in neighbouring coil.

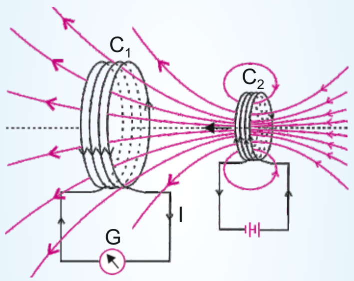
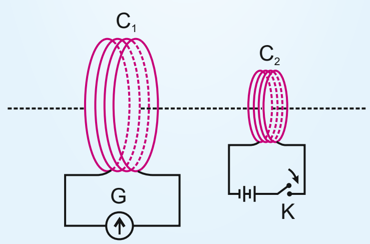


Fig. : Current is induced in coil C_1 due to motion of the current carrying coil C_2.

(c) Third experiment : Galvanometer shows a momentarily deflection when tapping key K is pressed



2 MAGNETIC FLUX

Magnetic flux through a surface of area $\vec{A}$ placed in uniform magnetic field $\vec{B}$ is written as $\phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$

For non-uniform magnetic field

$$\phi = \int \vec{B}\cdot d\vec{A}$$

3 FARADAY'S LAWS OF INDUCTION

Conclusion of experiments was formulation of laws:

(1) The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

(2) Mathematically the emf induced is given by

$$\varepsilon = -\frac{d\phi_B}{dt}$$

- Negative sign indicates the direction of ε and hence the direction of current in the closed loop.
- If loop contains N turns, change of flux is associated with each turn.

$$\varepsilon = -N\left(\frac{d\phi_B}{dt}\right)$$

- The induced emf can be increased by increasing the number of turns of closed coil.

4 LENZ'S LAW

- **LENZ'S LAW:** This law gives the polarity of induced emf. The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.
- The law is in accordance with the law of conservation of energy.

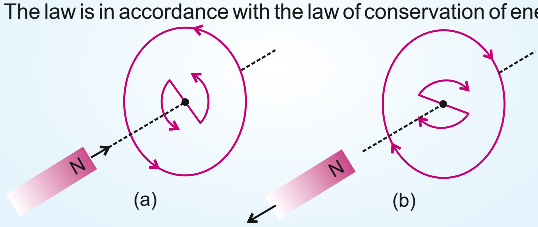
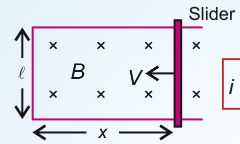


Fig. : Illustration of Lenz's law.

5 MOTIONAL EMF

(1) Straight conductor in motion: In uniform and time independent magnetic field.

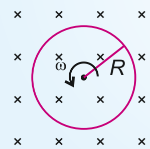


$e = Blv$ (emf across rod)

$$i = \frac{\varepsilon}{R} = \frac{Blv}{R} \qquad \Delta Q = \frac{\Delta\phi_B}{R} \qquad P = \frac{B^2l^2v^2}{R}$$

- Mechanical energy which is needed to move arm is converted into electric energy and then to thermal energy.

(2) Rod rotated about one end :



$$\varepsilon = \frac{B\omega R^2}{2}$$

Fleming's Right hand Rule : This gives the direction of induced emf or current in a conductor moving in a magnetic field. If we stretch forefinger, central finger and thumb of our right hand in mutually perpendicular directions such that forefinger along field, thumb along direction of motion of conductor then central finger will give the direction of induced current.

6 EDDY CURRENTS

Electric currents are induced in well defined path in a conductor like circular loops, when bulk piece of conductor is subjected to changing magnetic flux, induced currents are produced in them known as eddy currents.

The eddy currents are also called Foucault currents after its discovery.

- The changing magnetic flux induces current.
- These currents are used to advantage in many applications.
  (1) Magnetic braking of trains
  (2) Electromagnetic damping
  (3) Induction furnace
- Eddy currents dissipate energy in the form of heat energy.
- Eddy currents are minimized using laminations of metal to make a metal core

7 INDUCTANCE

The current can be induced in a coil by the flux change produced by same coil OR another coil.

- In both cases, flux through a coil is proportional to current

$$\frac{d\phi_B}{dt} \propto \frac{dI}{dt}$$

- Constant of proportionality is called inductance.
- Inductance is the ratio of flux linkage and current.
- This inductance depends on geometry of the coil and intrinsic material properties.

SELF INDUCTANCE

Phenomenon of induced EMF in a single isolated coil due to changing flux through the coil by means of varying the current through same coil is self induction.

Total flux linkage = Li

L is called self inductance.

$$\varepsilon = -L\frac{di}{dt}$$

- Self induced emf always opposes any change of current in the coil.

  Self inductance of a solenoid is $L = \mu_0 n^2 Al$

  n is number of turns per meter of solenoid length. When solenoid is filled with some material

$$L = \mu_0\mu_r n^2 Al$$

- Self inductance plays the role of inertia. It is electromagnetic analogue of mass in mechanics.
- Unit of self inductance is henry (H) in SI units.
- Self inductance of the coil depends on its geometry and on the permeability of the medium.

MUTUAL INDUCTANCE

- Varying current in one coil can induce emf in neighbouring coil.

$$\varepsilon_1 = M\frac{di_2}{dt}$$

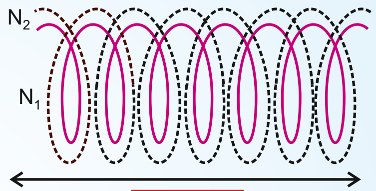
The magnitude of induced emf depends on rate of change of current and mutual inductance of two coils:

- SI unit of inductance is Henry and is denoted by H. Its dimensional formula is $ML^2T^{-2}A^{-2}$.

For two long co-axial solenoids each of length l

$$M_{12} = \mu_0 n_1 n_2 Al$$

M_{12} is coefficient of mutual induction



$$M_{12} = M_{21}$$

- Mutual inductance of a pair of coils, solenoids depends on their separation as well as their relative orientations.
- For two concentric circular coils with radius r and R ($R >> r$) coils are coplanar also.

$$M_{12} = M_{21} = \frac{\mu_0\pi(r)^2}{2R}$$

8 MAGNETIC POTENTIAL ENERGY

- Energy required to built any current I in a system of self inductance L

$$W = \frac{1}{2} \times L \times I^2$$

- This work done gets stored as magnetic potential energy.

$$U_B = \frac{1}{2}LI^2 = \frac{B^2A\ell}{2\mu_0}$$

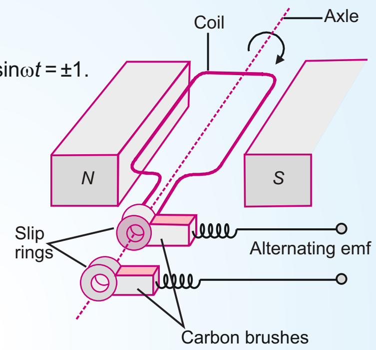
- Magnetic energy per unit volume.

$$u_B = \frac{B^2}{2\mu_0}$$ → Energy is proportional to square of field strength

9 AC GENERATOR

- This technology is based on electromagnetic induction phenomenon.
  - Modern A.C. generator has output capacity upto 100 MW.
  - This machine converts mechanical energy into electric energy.
  - The emf induced is sinusoidal.

$$\varepsilon = NBA\omega \sin\omega t$$



$NBA\omega$ is the maximum value of emf when $\sin\omega t = \pm 1$.

$$\varepsilon_0 = NBA\omega \qquad \varepsilon = \varepsilon_0 \sin\omega t$$

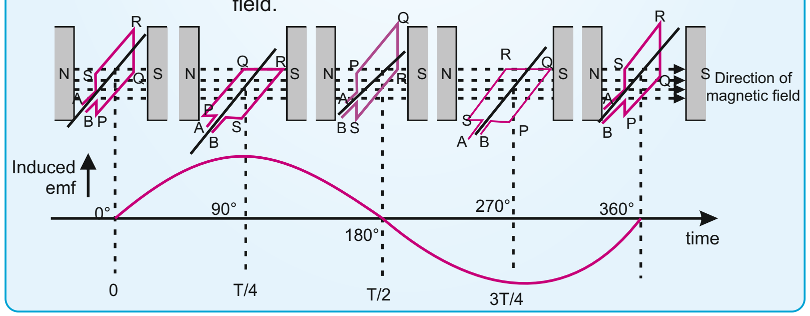
ω is angular speed of rotor of ac generator.

The direction of current and emf changes periodic with time

$$\varepsilon = \varepsilon_0 \sin(2\pi\nu t)$$

- ν in India is 50Hz
- ν in USA is 60Hz

Stage-1	Stage-2	Stage-3	Stage-4	Stage-5
The plane of the armature is perpendicular to the magnetic field.	When the armature rotates about through 90° the plane of the armature is parallel to magnetic field.	Armature after a rotation of 180°	Armature after a rotation of 270°	Armature after a rotation through 360°



Alternating Current

Chapter 7

1 ALTERNATING CURRENT

Current which changes continuously in magnitude and periodically in direction.

2 ROOT MEAN SQUARE VALUE

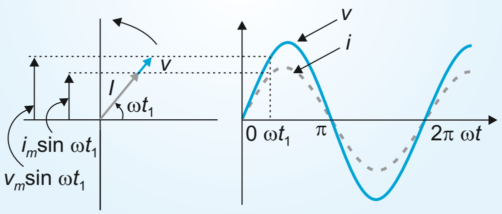
r.m.s. value of a function over a period T is given by $\sqrt{\dfrac{\int f(t)^2 dt}{\int dt}}$

- RMS current is equivalent to *dc* current that would produce same average power loss as alternating current.

3 AVERAGE VALUE

Average value of a function over a period T is given by $\langle f(t) \rangle = \dfrac{1}{T}\int_0^T f(t)dt$

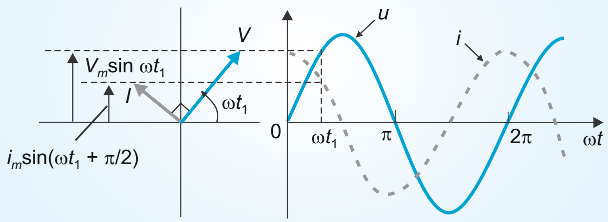
4 PURELY RESISTIVE CIRCUIT

- Let applied voltage is $\varepsilon = \varepsilon_0 \sin \omega t$
- Alternating voltage is in phase with current
- $I = \dfrac{\varepsilon}{R} = I_0 \sin\omega t$
- $I_0 = \dfrac{\varepsilon_0}{R}$
- $P_{av} = \dfrac{1}{2} I_0^2 R$
- 

(a) (b)

5 PURELY CAPACITIVE CIRCUIT

- Let applied voltage is $\varepsilon = \varepsilon_0 \sin \omega t$
- Current leads the voltage by a phase angle $\pi/2$
- $I = I_0 \sin\left(\omega t + \dfrac{\pi}{2}\right)$
- $I_0 = \dfrac{\varepsilon_0}{X_C} = \omega C \varepsilon_0$, where $X_C = \dfrac{1}{\omega C}$

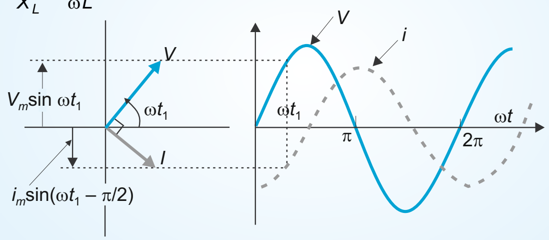


(a) (b)

- Instantaneous power $= \dfrac{\varepsilon_0 I_0}{2}\sin[2\omega t]$
- Average power = 0

6 PURELY INDUCTIVE CIRCUIT

- Let applied voltage is $\varepsilon = \varepsilon_0 \sin \omega t$
- Current lags behind the voltage by a phase angle $\pi/2$
- $I = I_0 \sin(\omega t - \pi/2)$
- Current reaches maximum value later than voltage by one fourth of period.
- $I_0 = \dfrac{\varepsilon_0}{X_L} = \dfrac{\varepsilon_0}{\omega L}$, where $X_L = \omega L$



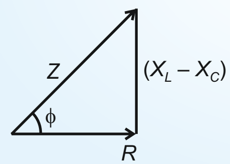
(a) (b)

- Instantaneous power supplied $= -\dfrac{I_0 V_m}{2}\sin 2\omega t$
- Average power supplied by an inductor over one complete cycle is zero.

7 SERIES LCR CIRCUIT

- $\varepsilon = \varepsilon_0 \sin \omega t$ and $I = I_0 \sin(\omega t - \phi)$
- $I_0 = \dfrac{\varepsilon_0}{z}$, where $z = \sqrt{R^2 + (X_L - X_C)^2}$, $\varepsilon_0^2 = (V_{Ro})^2 + (V_{Co} - V_{Lo})^2$
- Phase difference between current and voltage is ϕ, $\tan\phi = \dfrac{X_L - X_C}{R}$
- If $X_L > X_C \Rightarrow \phi$ is +ve (Inductive)
- If $X_L < X_C \Rightarrow \phi$ is –ve (Capacitive)
- If $X_L = X_C \Rightarrow \phi = 0$ (Resistive)

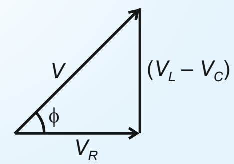
Impedance and Voltage Triangles



Impedance Triangle

$\tan\phi = \dfrac{V_{Lo} - C_{Co}}{V_{Ro}}$

$\tan\phi = \dfrac{X_L - X_C}{R}$



Voltage Triangle

8 POWER IN AC CIRCUITS

$P_{avg} = \varepsilon_{rms} I_{rms} \cos\phi = \dfrac{\varepsilon_0 I_0}{2}\cos\phi = I_{RMS}^2 Z\cos\phi$

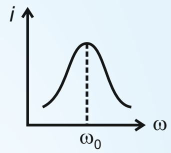
$= I_{rms}^2 R = \left(\dfrac{\varepsilon_{rms}^2}{z^2}\right) R$

9 POWER FACTOR

- Power factor: $\cos\phi = \dfrac{R}{Z}$
- In pure resistive circuit
  $\phi = 0° \Rightarrow \cos\phi = 1$
- In pure inductive circuit or pure capacitive circuit
  $\phi = \pm\dfrac{\pi}{2} \Rightarrow \cos\phi = 0$
- In series LCR circuit at resonance, $X_L = X_C$
  $\Rightarrow$ $Z = R$ and $\phi = 0°$
  $\Rightarrow$ $\cos\phi = 1$ (POWER FACTOR)

10 RESONANCE IN LCR SERIES CIRCUIT

- When $X_L = X_C \Rightarrow Z = R$
- Current become maximum
- $i_0 = \frac{V_0}{R}$



- $\omega_0 = \frac{1}{\sqrt{LC}}$
- $\Delta\omega = \frac{R}{2L}$

  where $\omega_1 = \omega_0 + \Delta\omega$

  $\omega_2 = \omega_0 - \Delta\omega$
- **Quality factor**

  It is a measure of sharpness of resonance

  $$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\text{Resonance frequency}}{\text{Band width}}$$

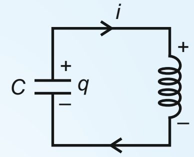
  $$= \frac{\omega_0 L}{R}$$

  $$= \frac{1}{\omega_0 RC}$$

  $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$
- Resonance circuits are used in tuning mechanism of radio or TV set.
- Resonance phenomenon is exhibited by a circuit having both L and C present in circuit.
- If resonance is not sharp, maximum current is less, the circuit is close to resonance for a larger range of $\Delta\omega$ of frequencies and tuning of circuit will not be good. For good quality L be large and R low.

11 LC OSCILLATIONS

- $\frac{d^2q}{dt^2} + \frac{1}{LC}(q) = 0$
- $q = q_0 \cos(\omega_0 t + \phi)$
- $\omega_0 = \frac{1}{\sqrt{LC}}$



- $U_{\text{Total}} = \frac{q_m^2}{2C} = \frac{1}{2}Li_m^2$

Analogies Between Mechanical and Electrical Quantites

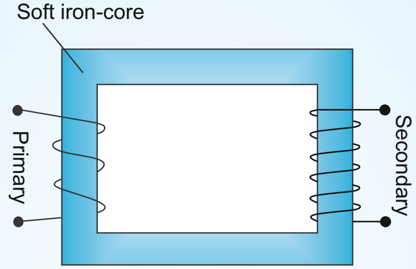
Mechanical system	Electrical (system)
Mass (m)	Inductance (L)
Force constant (K)	Reciprocal capacitance $\left(\frac{1}{C}\right)$
Displacement (x)	Charge (q)
Velocity $\left(v = \frac{dx}{dt}\right)$	Current $\left(i = \frac{dq}{dt}\right)$
Mechanical energy $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$	Electromagnetic energy $U = \frac{q^2}{2C} + \frac{1}{2}Li^2$

- Every inductor has resistance. This introduces damping effect on charge and current and oscillation finally die away.
- Total energy is radiated away from the system in the form of electromagnetic waves. The Radio and TV transmission depend on this radiation.

12 TRANSFORMER

- To change an alternating voltage from one to another of greater or smaller value, we use transformer.

  A transformer consists of two sets of coils, insulated from each other. It works on mutual induction principle.



$$\frac{\varepsilon_P}{\varepsilon_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S} = \frac{\phi_P}{\phi_S} = \text{constant}$$

- Efficiency of transformer

  $$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\varepsilon_S I_S}{\varepsilon_P I_P}$$

Step-up Transformer	Step-down Transformer
Number of turns in primary binding is lesser than in secondary binding	Number of turns in secondary binding is lesser than in primary binding
$N_S > N_P$	$N_S < N_P$
$E_S > E_P$	$E_S < E_P$
$I_S < I_P$	$I_S > I_P$

- Output voltage of transformer is out of phase with input.
- Energy losses in a transformer are due to

  A. Flux Leakage

  B. Resistance of winding

  C. Eddy currents

  D. Hysteresis

Electromagnetic Waves

8
Chapter

1 MAXWELL'S EQUATIONS

$\oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_0}$ (Gauss's Law of electrostatics)

$\oint \vec{B}.d\vec{A} = 0$ (Gauss's Law of magnetism)

$\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$ (Faraday's Law of EMI)

$\oint \vec{B}.d\vec{l} = \mu_0\left(i_c + \varepsilon_0\frac{d\phi_E}{dt}\right)$ Maxwell-Ampere's circuital theorem

- These equations express all basic laws of electro-magnetism

2 DISPLACEMENT CURRENT

- Source of a magnetic field is not just the conduction electric current but also time varying rate of change of electric field.
- It arises whenever the electric flux is changing with time.

$i_d = \varepsilon_0\frac{d\phi_E}{dt}$

- Changing electric field must also produce a magnetic field.

3 ELECTROMAGNETIC WAVES

Sources of Waves

- Through accelerating charges
- Through oscillating electric dipoles
- By harmonically oscillating electric charges
- Oscillating fields of electromagnetic waves can accelerate charges and can produce oscillating currents
- Electromagnetic waves in interaction with matter interacts through its electric and magnetic field which set in oscillating charges present in all matter
- Mechanism of absorption and scattering depends on wavelength of electromagnetic wave, nature of atoms and molecules in medium

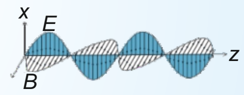
4 NATURE OF ELECTROMAGNETIC WAVES

- Waves are sinusoidal, having variation of electric and magnetic field at right angles to each other and perpendicular to direction of wave propogation.
- Frequency of electromagnetic wave equals the frequency of oscillating charge.
- Energy associated with propagating wave comes from energy of source, the accelerated charge.
- Produced and detected by Hertz in 1887 in laboratory. This verified a basic prediction of Maxwell's equation.
- No material medium is involved in vibration of electric and magnetic fields.
- E.M. wave carry energy and energy is shared equally by electric and magnetic fields.
- Electromagnetic wave other than light also have same velocity in vacuum.
- Wave transports momentum and exerts pressure

  $P = \frac{U}{c}$, (U = Energy transfer for complete absorption by surface)

- The magnitude of total momentum delivered.
- So wave can carry energy from one place to another.
- $E_x = E_0\sin(kz - \omega t)$ and $B_0 = E_0/c$

  $By = B_0\sin(kz - \omega t)$ (wave equations)



- k = wave vector and speed of propagation is, $v = \frac{\omega}{k}$

  in medium, speed of wave, $v = \frac{1}{\sqrt{\mu\varepsilon}}$

  In vacuum, $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

- This is fundamental constant. These waves can be polarized.
- Energy density in E field, $u_E = \frac{1}{2}\varepsilon_0 E^2$
- Energy density in B field, $u_B = \frac{B^2}{2\mu_0}$
- $u_E = u_B$
- $I = \varepsilon_0 E_{rm}^2 c$

ORDER OF FREQUENCY

- Gamma rays > X rays > U.V > Visible > Infrared > microwaves > short radio waves > FM and TV > AM Radio > Long Radio waves

5 SPECTRUM OF ELECTRO-MAGNETIC WAVES AND PROPERTIES

A. RADIO WAVES

- Wavelength greatest
- Produced by vibrating electrons
- AM BAND : 530 kHz to 1710 kHz ,
- SHORT band: 3 MHz to 54 MHz
- FM band : 88 MHz to 108 MHz
- UHF band : CELULAR PHONE

B. MICRO WAVES

- Wavelength 0.1 m to 1 mm (Frequency GHz)
- Produced by special vacuum tubes (Klystrons, Magnetrons and Gunn diodes)
- Used in radar system
- Micro wave oven works with microwaves
- Used in speed guns

C. INFRARED WAVES

- Wavelength 1 mm to 700 nm
- Produced in vibration of atoms and molecules
- These are called heat waves
- Used in physical therapy
- Trapped by greenhouse gases
- Remote switches
- Infrared detector

D. VISIBLE RAYS

- Wave length 700 nm to 400 nm from electrons in atom by transitions from high to lower level
- Detected by human eye
- Different animals are sensitive to different ranges of wave length

E. UV RAYS

- Wavelength 400 nm to 1 nm
- Sun is import ant source
- Absorbed by glass used by welders
- Used in eye surgery (LASIK)
- UV lamps for water purefication

F. X-RAY

- Wavelength 10 nm to 10^{-4} nm
- Obtained from bombarding a metal target by high energy electron
- Used as diagnostic tools to treat cancer.
- Photographic films

G. γ-RAYS

- Wavelength 10^{-10} m to 10^{-14} m
- Produced in nuclear reaction
- Emitted by radioactive nuclei
- used in medicine to destory cancer cells

Ray Optics and Optical Instruments

9
Chapter

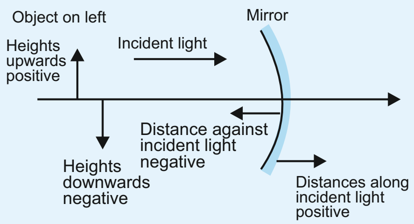
1 REFLECTION OF LIGHT

Law of Reflection

- Incident ray, reflected ray and normal to reflecting surface at the point of incidence lie in the same plane.
- Angle of incidence is equal to angle of reflection.

Sign-convention

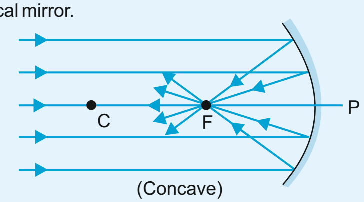
- In sign convention, all distances measured in the same direction as incidence ray are taken positive and those measured in the direction opposite of incident ray are taken negative.
- The heights taken above the principal axis are positive and below negative.



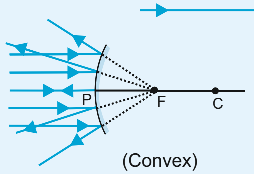
Focal Length of Spherical Mirrors

- The distance between focus and pole of a mirror is called focal length.
- Focal length is equal to half of radius of curvature of the curved spherical mirror.

$$f = \frac{R}{2}$$



(Concave)



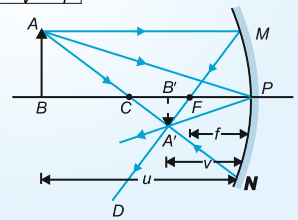
(Convex)

Image Formation by Spherical Mirrors

- The image by a mirror is real if rays after reflection actually meet and virtual if rays are not actually meeting but appear to diverge from a point.
- An incident ray passing through centre of curvature of mirror retraces its path.
- Mirror equation is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ and magnification formula

$$m = -\frac{v}{u} = \frac{f}{f-u}$$

$$= \frac{f-v}{f}$$



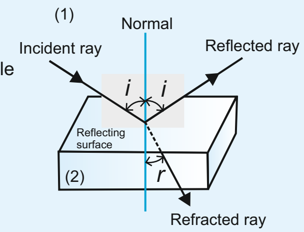
2 REFRACTION OF LIGHT

- When a beam of light encounters another transparent medium, part of light is reflected back. This called internal reflection. The rest of light enter other medium.
- When light is incident obliquely, its propagation direction changes in other medium, this phenomenon is called refraction.
- Red light travels faster than blue light in same medium.

Law of Refraction

- The incident ray, refracted ray and normal to interface at the point of incidence, all lie in same plane.
- The ratio of sine of angle of incidence to the sine of angle of refraction is constant.

$$\frac{\sin i}{\sin r} = n_{21}$$

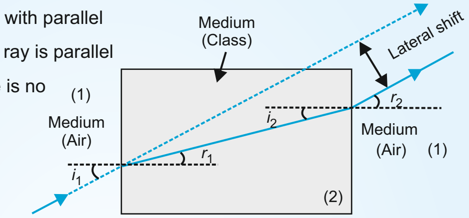


n_{21} is refractive index of second medium with respect to first.

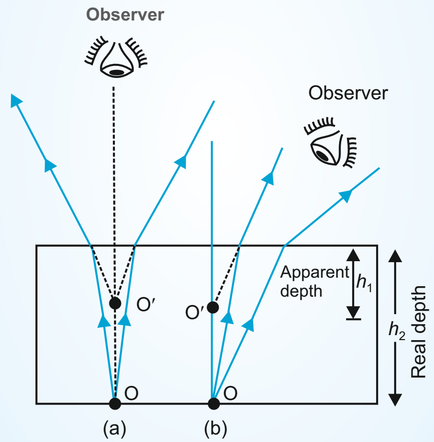
- Optical denser medium has high refractive index. Mass density of optical denser medium may be less than mass density of rarer medium.
- Elementary results from laws of refraction are

(1) $n_{32} = n_{31} \times n_{12}$

(2) For rectangular slab with parallel faces, the emergent ray is parallel to incident ray, there is no deviation but has lateral shift.

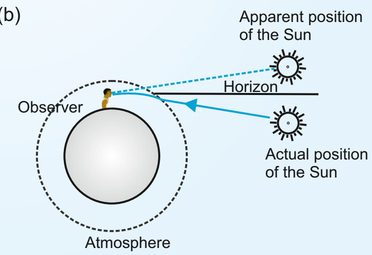


(3) Bottom of tank filled with water appears to be raised.



$$\text{Apparent depth} = \frac{\text{Real depth}}{n_{21}}$$

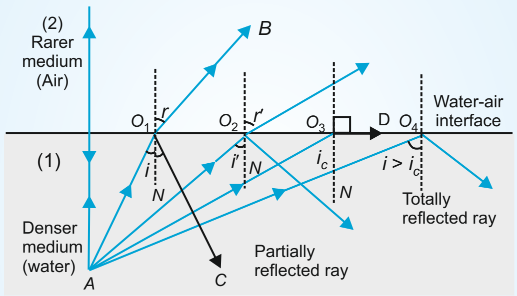
(4) Sun is visible a little before the actual sunrise and until a little after the sunset, this time difference is about 2 minute, the sun appears oval shaped.



3 TOTAL INTERNAL REFLECTION

- If angle of incidence, for light traveling from denser to rarer medium is greater than certain angle called critical angle for the media, no light is transmitted.

$\sin i_c = \frac{1}{n_{12}}$ n_{12}: refractive index of denser medium w.r.t rarer medium.



- Higher is value of refractive index, smaller will be critical angle.

Substance	Ref. index	Critical angle
Water	1.33	48.75°
Flint glass	1.62	37.31°
Diamond	2.42	24.41°

Phenomenon based on TIR are
- Mirage
- Sparkling of diamond
- Special prisms of flint glass to bend light by 90° and 180°
- Optical fibre for communication

7 THIN LENSES IN CONTACT

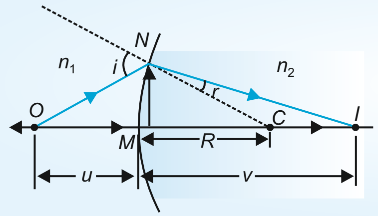
- When thin lenses are kept in contact

$P = P_1 + P_2 + P_3 + ...$

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...$

- Net power is individual power's algebraic sum. Some terms may be positive (convex) and some terms may be negative (concave) on right hand side.
- This combination helps to get diverging or converging lens combination of desired magnification.

4 REFRACTION AT SPHERICAL SURFACES



$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

R = radius of curvature of the curved spherical surface. It holds for any curved surface (for paraxial approximation).

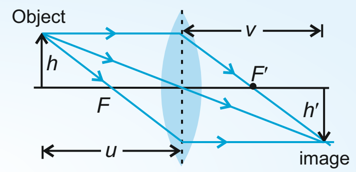
5 REFRACTION BY LENSES

- A lens is a transparent optical medium bounded by two surfaces.
  At least one surface should be spherical.
- After two refraction through a lens, image is formed.
  The thin lens formula becomes

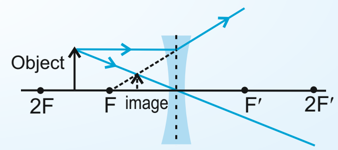
$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$m = \frac{\text{Size of image}}{\text{Size of object}} = \frac{h'}{h} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$

- Formula is valid for convex and concave lenses and magnification produced by lens (for paraxial approximation)



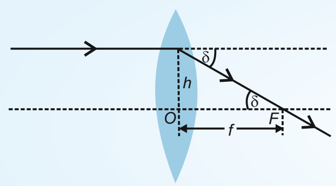
(a)



(b)

6 POWER OF THIN LENS

- It is measure of its convergence or divergence ability.



- The power P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light parallel to the principal axis falling at unit distance from the optical centre.
- A lens of shorter focal length bends incident ray more and has high power.

$P = \frac{1}{f}$

- Its SI unit is dioptre (D)

$1\text{D} = 1\ \text{m}^{-1}$

- It is positive for converging lens and negative for diverging lens.

Lens maker's formula

$P = \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

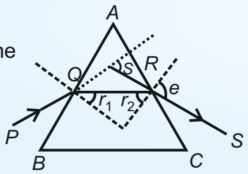
- n is relative refractive index of glass with respect to surrounding and R_1 and R_2 are radii of curvature of two surfaces.
- A converging lens in a transparent liquid of refractive index greater than lens glass behaves like a diverging lens and vice versa.

8 REFRACTION THROUGH A PRISM

- For any triangular prism angle between incidence ray and emergent ray is called angle of deviation

$\delta = i + e - A$

- δ remains same if i and e are interchanged.

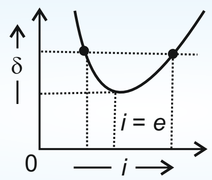


When $\delta = D_m$

$i = e, \quad D_m = 2i - A$

$r = r_1 = r_2$ or $r_1 = A/2$

The refracted ray inside prism becomes parallel to its base.



- Refractive index of prism is calculated by formula.

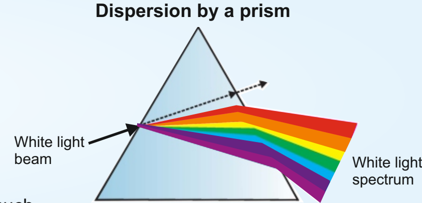
$$n_{21} = \frac{n_2}{n_1} = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin (A/2)}$$

- For small angle thin prism

$D_m = (n_{21} - 1)A$

- It implies thin prism don't deviate light much

Dispersion by a prism

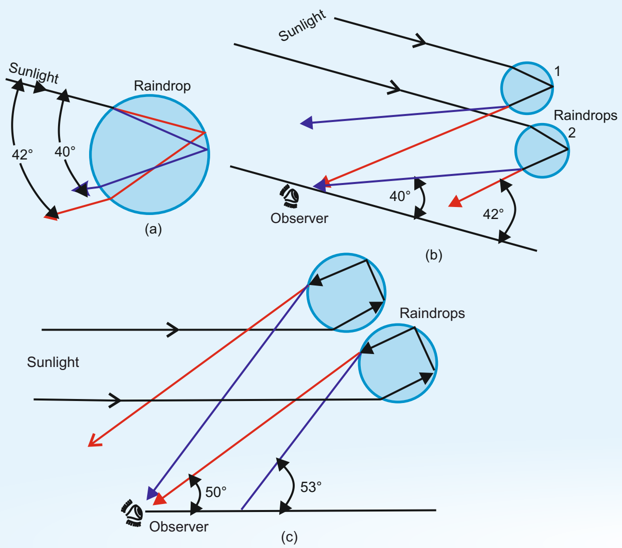


9 NATURAL PHENOMENA DUE TO SUNLIGHT

Dispersion of Light

- The phenomenon of splitting of light into constituent colours is known as dispersion.
- Dispersion takes place due to different refractive index of medium for different wavelengths.
- Chromatic aberration in thick lenses is due to dispersion.
- Rainbow is an example of dispersion of light (sun) by water drops
- In vacuum speed of light is independent of wavelength. So Vacuum is non dispersive medium.

Rainbow



Rainbow: (a) The sun rays incident on a water drop get refracted twice and reflected internally by a drop; (b) Enlarge view of internal reflection and refraction of a ray of light inside a drop form primary rainbow; and (c) Secondary rainbow is formed by rays undergoing internal reflection twice inside the drop.

10 OPTICAL INSTRUMENTS

Eye Defects Cure

- Myopia → Concave lens
- Hypermetropia → Convex lens
- Astigmatism → Cylindrical lens

Simple Microscope

- Microscope : A simple magnifier or microscope is a converging lens of high power.
- Angular magnification is equal to ratio of angular size of image to angular size of object
- Final image at near point

$$m = \left[1 + \frac{D}{f}\right]$$

- Final image at infinity

$$m = \frac{D}{f}$$

Compound Microscope

- For large magnification objective and eye piece should have low focal length
- Final image at near point

$$m = m_o \times m_e = \frac{v_o}{u_o}\left[1 + \frac{D}{f_e}\right]$$

- Final image at infinity

$$m = \frac{v_o}{u_o}\left(\frac{D}{f_e}\right)$$

Telescope

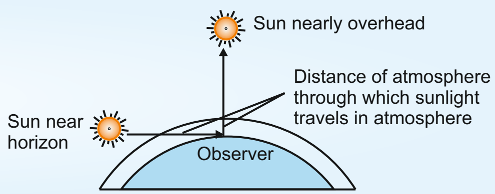
- It is used to provide angular magnification for distant objects
- Final image at infinity

$$m = \frac{f_o}{f_e} \text{ and } L = f_o + f_e$$

- Final image at near point

$$m = \frac{f_o}{f_e}\left[1 + \frac{f_e}{D}\right] \text{ and } L = f_o + \frac{Df_e}{D + f_e}$$

Scattering of light: Light of shorter wavelength is scattered much more than of longer wavelength. Amount of scattering is inversely proportional to fourth power of wavelength.



Wave Optics

10
Chapter

1 HUYGEN'S PRINCIPLE

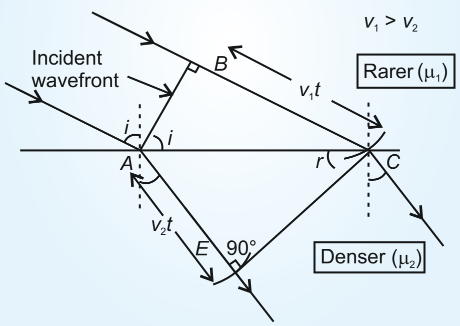
- Huygen gave a geometrical method for the propagation of wave in any medium.
- **Wavefront :** Surface of constant phase. The line drawn perpendicular to wavefront gives direction of propagation of wave and energy.
- Each point on primary wavefront behaves like a new wave source from which secondary waves emit in all directions.
- If we draw the envelope of these secondary wavelets then it will give the position of secondary wavefront.
- The shape of wavefront depends on shape of wave source.
- Point source - Spherical wave fronts
- Line source - Cylindrical wavefronts
  at a large distance from the source, a small portion of wavefront is planar.

2 REFRACTION OF PLANE WAVEFRONT

From ΔABC, $\sin i = \dfrac{BC}{AC}$

From ΔAEC, $\sin r = \dfrac{AE}{AC}$

$$\frac{\sin i}{\sin r} = \frac{BC}{AE} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$$



3 REFLECTION OF A PLANE WAVEFRONT AT PLANE SURFACE

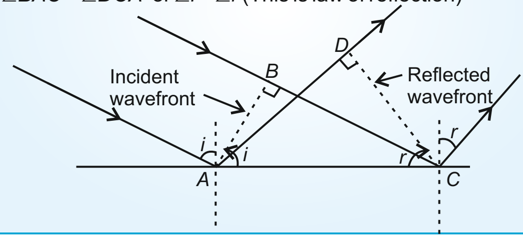
From ΔABC and ΔACD, $BC = AD = vt$

$\angle ABC = \angle ADC = 90°$

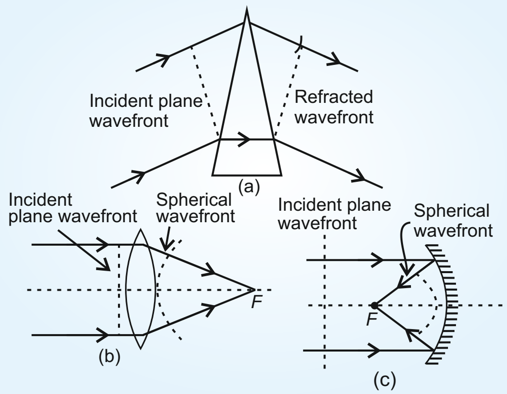
AC is common.

So ΔABC and ΔACD are congruent

$\therefore \angle BAC = \angle DCA$ or $\angle i = \angle r$ (This is law of reflection)



4 REFRACTION OF PLANE WAVE BY PRISM, LENS AND MIRROR



5 THE DOPPLER'S EFFECT

When the source moves away from observer the frequency as measured by source will be smaller and wavelength will be longer, this is called red shift. Towards the observer, there is an apparent decrease in wavelength, this is called red shift. $\dfrac{\Delta\nu}{\nu} = -\dfrac{v_{radial}}{C}$

6 COHERENT SOURCE

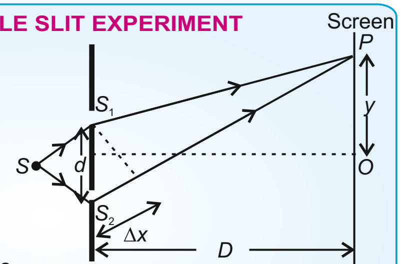
- If the phase difference between two waves reaching at a point remains constants with time, then the sources are said to be coherent.
- When the phase difference between two waves change with time it is incoherent.

7 PRINCIPLE OF SUPERPOSITION

- If number of waves reach at a point, then the resultant displacement of point is the vector sum of displacement of individual waves at that point and at that time.
- Consider two waves reach at origin $y_1 = a_1\cos\omega t$, $y_2 = a_2\cos(\omega t + \phi)$
  From superposition law resultant amplitude is $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$
- **For A_{max} or constructive interference**
  Phase difference, $\phi = 0, 2\pi, 4\pi \ldots 2n\pi$
  Path difference, $\Delta x = \lambda, 2\lambda, \ldots n\lambda$ where $n = 0, 1, 2, 3 \ldots$
  $A_{max} = (a_1 + a_2)$, $I_{max} \propto (a_1 + a_2)^2$
  For A_{min} or destructive interference
  Phase difference, $\phi = \pi, 3\pi, 5\pi \ldots (2n+1)\pi$
  Path difference, $\Delta x = \lambda/2, 3\lambda/2 \ldots, (2n+1)\lambda/2$ where $n = 0, 1, 2, 3, \ldots$
  $A_{min} = (a_1 - a_2)$, $I_{min} \propto (a_1 - a_2)^2$
  - If $a_1 = a_2 = a$, $A = 2a\cos(\phi/2)$ and $I_1 = I_2 = I_0 \Rightarrow I = 4I_0\cos^2(\phi/2)$
  - When phase difference between two vibrating sources changes rapidly with time, two sources are incoherent and the intensities just add up. i.e. $I = I_1 + I_2$

8 YOUNG'S DOUBLE SLIT EXPERIMENT

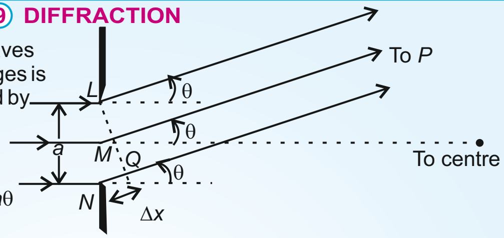
- Path difference,
  $\Delta x = S_2P - S_1P$, $\Delta x = yd/D$
- For constructive interference or bright fringes
  $yd/D = n\lambda$, $n = 0 \pm 1, \pm 2, \ldots$
- For destructive interference or Dark fringes
  $yd/D = (2n+1)\lambda/2$, $n = 0 \pm 1, \pm 2, \ldots$



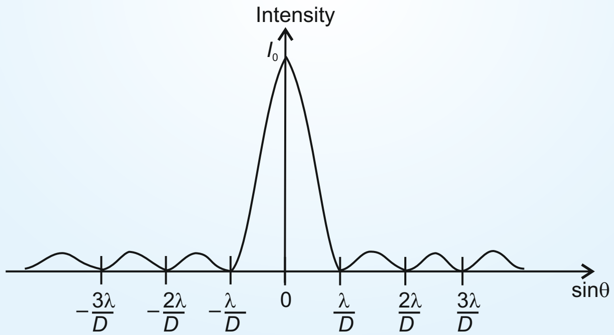
- Distance between two consecutive bright (or dark) fringe called fringe width(β)
  $\beta = x_{n+1} - x_n = \lambda D/d$
- The fringe pattern is hyperbolic, for large distances the fringe will be nearly straight lines.

9 DIFFRACTION

- The phenomena of bending of waves at the narrow holes and sharp edges is called diffraction. This is exhibited by all types of waves.
- **Single slit diffraction**
  Path difference at point P
  $\Delta x = NP - LP = NQ$; $\Delta x = a\sin\theta \simeq a\theta$

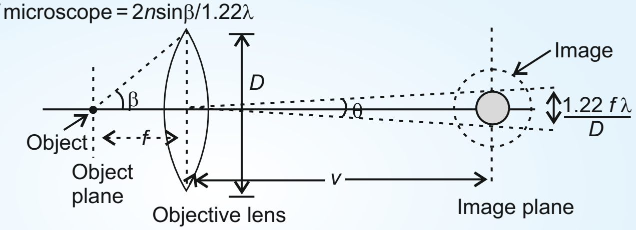


- **For central maxima** $\theta = 0$
- **For secondary maxima :** $\theta \simeq (n + 1/2)\lambda/a$ where $n = \pm1, \pm2, \pm3, ...$
- **For minima :** $\theta \simeq n\lambda/a$ where $n = \pm1, \pm2, \pm3, ...$
- Width of central fringe $W_c = 2\lambda D/a$
- Width of secondary fringe $W = \lambda D/a$

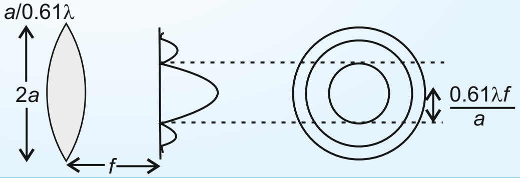


10 RESOLVING POWER OF OPTICAL INSTRUMENTS

- It is the ability to resolve the image of two nearby objects distinctly.
- Resolving power of microscope $= 2n\sin\beta/1.22\lambda$



- The product $n\sin\beta$ is called the numerical aperture.
- Resolving power of telescope $= a/0.61\lambda$



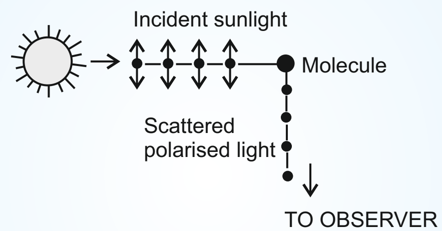
11 VALIDITY OF RAY OPTICS

Fresnel's Distance :

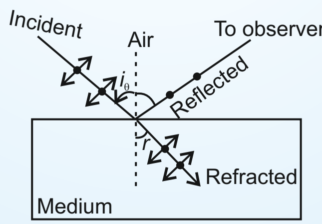
- Distance at which diffraction spread is equal to the size of aperture. $z_f \simeq a^2/\lambda$
- This gives the distance beyond which divergence of beam of width a becomes significant.
- Ray optics is valid in the limit of wavelength tending to zero.

12 POLARISATION

- The phenomena of restricting the vibration of electric vector only in one direction perpendicular to the direction of propagation is called polarisation.
- **Malus' law :** The intensity of transmitted light passed through an analyser is $I = I_0\cos^2\theta$
  where, I_0 = Intensity of polarised light after passing through polariser.
  θ = Angle between axis of polariser and analyser
- **Polarisation by scattering :** An observer looking at 90° to the direction of sun. The radiations scattered by the molecules perpendicular to the incident unpolarised light (figure) is polarised.
- Scattering of light by molecules studied by C.V. Raman is called Raman effect.



- **Polarisation by Reflection :** When an unpolarised light is incident at Brewster's angle (i_B), then reflected light is polarised and refracted light is partially polarised. Brewster's angle depends on refractive index of two media,
  $\mu = \tan i_B$ and $i_B + r = 90°$



- Electric field vector perpendicular to plane of incidence is reflected.

Dual Nature of Radiation and Matter

11
Chapter

1 ELECTRON EMISSION

Thermionic Emission

- The process of emission of electrons when a metal is heated is known as thermionic emission
- The emitted electrons are called thermions
- Emitted number of thermions depends on temperature of metal surface

Field Emission

- The process of emission of free electrons when a strong electric field ($\approx 10^8$ V/m) is applied across the metal surface is called field emission or cold emission, as in spark plug.

Photoelectric Emission

- The process of emission of electrons when light of suitable frequency is incident on metal surface is called as photoelectric emission
- Emitted electrons are called photoelectrons
- Number of photoelectrons emitted depends on the intensity of incident light

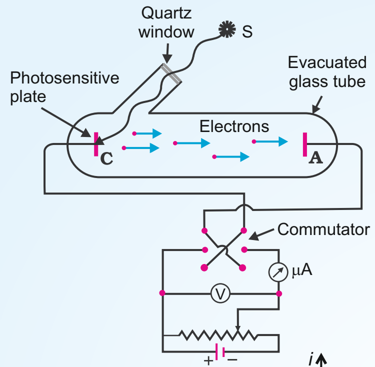
Secondary Emission

- The process of emission of free electrons when highly energetic electron beam is incident on a metal surface is called secondary emission.
- The emitted electron is called secondary electrons.

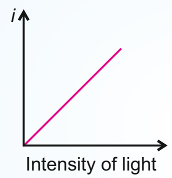
2 PHOTOELECTRIC EFFECT

- The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz
- Wilhelm Hallwachs and Philipp Lenard investigated the phenomenon of photoelectric emission in detail during 1886-1902.
- Certain metals like zinc, cadmium, magnesium etc responded only to ultraviolet light to cause electron emission. However, some alkali metals such as Lithium, Sodium, Potassium, Caesium and rubidium were sensitive to visible light.

3 EXPERIMENTAL STUDY OF PHOTOELECTRIC EFFECT



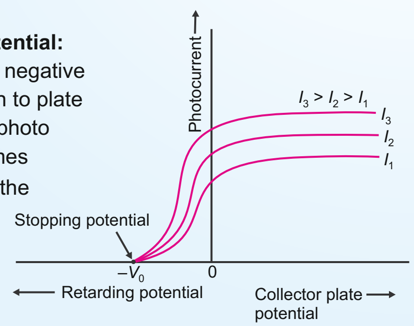
1. **Effect of Intensity:** The number of photoelectrons emitted per second or photoelectirc current is directly proportional to the intensity of radiation.



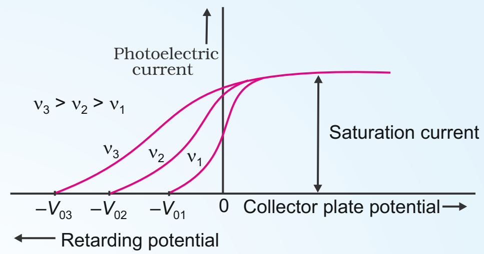
2. **Effect of potential on photoelectric current:**
   (i) For a given frequency of incident radiation, stopping potential is independent of intensity.
   (ii) Maximum kinetic energy

   $K_{max} = eV_0$

- **Stopping potential:** The minimum negative potential given to plate for which the photo current becomes zero is called the cut-off or stopping potential.



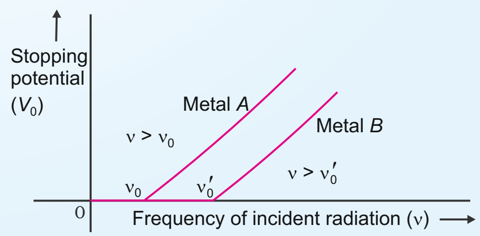
3. **Effect of frequency of incident radiation on stopping potential:**
   (i) Saturation current is independent of frequency
   (ii) Stopping potential depends on frequency of radiation.
   $(h\nu - h\nu_0 = eV_0)$



- The maximum kinetic energy of photoelectrons varies linearly with the frequency of incident radiation, but is independent of intensity.
- For a frequency less than threshold ($\nu < \nu_0$) no photoelectric emission is possible even if intensity is large.
- **Threshold frequency (ν_0):** The minimum frequency of incident radiation required to emit electrons called threshold frequency. It is different for different metals.
- **Work function (ϕ_0):** The minimum energy of incident radiation required to emit electrons from metal called work function

  $\phi_0 = h\nu_0$

- Emission starts in a time of the order of 10^{-9} s or less.



4 EINSTEIN'S PHOTOELECTRIC EQUATION: ENERGY QUANTUM OF RADIATION

In 1905 Albert Einstein proposed that radiation energy is built up of discrete units-the so called quanta of energy radiation. Later it was called photon. Each photon has energy ($h\nu$). When energy of photon striking at surface is greater than work function (ϕ_0), electron is emitted

Maximum kinetic energy of electrons

$K_{max} = eV_0 = h\nu - \phi_0$ h = Planck's constant = 6.626×10^{-34} Js

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e}$$

This is equation of straight line with slope $\left(\frac{h}{e}\right)$

- $\left(\frac{h}{e}\right)$ is independent of nature of material
- All photons of frequency (ν), have the same energy ($h\nu$), momentum, $P = \left(\frac{h}{\lambda}\right)$
- Photons are electrically neutral
- Increase in intensity of a given frequency means increase in number of photons per second crossing a given area.

5 PHOTOCELL

- It is a device which converts light energy into electrical energy.
- The photocurrent produced is of order of microampere.
- In gas filled photocell, current is increased due to ionization of the gas.
- It is also called electronic eye
- It is used in operation of control system and in light measuring devices

6 WAVE NATURE OF MATTER

de-Broglie proposed that the wavelength λ associated with a particle of momentum P is

$$\lambda = \frac{h}{P} = \frac{h}{mv} = \frac{h}{\sqrt{2Km}}$$

- If a charged particle having charge q accelerate from rest through a potential V.

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

(i) For electron, $\lambda = \frac{1.227}{\sqrt{V}}$ nm.

(ii) For proton, $\lambda = \frac{0.0286}{\sqrt{V}}$ nm.

(iii) For α-particle, $\lambda = \frac{0.0101}{\sqrt{V}}$ nm.

- According to Heisenberg, it is not possible to measure both the position and momentum of a particle at the same time exactly.
- $\Delta x \Delta p \approx \frac{h}{2\pi}$

8 ELECTRON MICROSCOPE

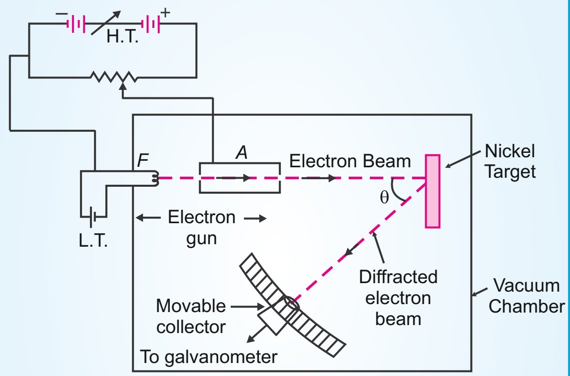
- It is a practical device that relies the wave characteristics of electron.
- Resolving power of electron microscope $\propto \frac{1}{\lambda}$

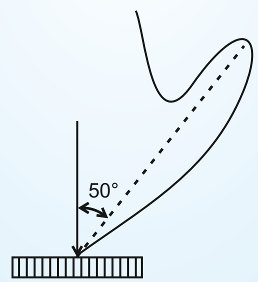
$$\propto \sqrt{V} \quad (V \text{ is accelerating potential})$$

- Resolving power of electron microscope is approximately 10^5 times the resolving power of optical microscope.

7 DAVISSON AND GERMER EXPERIMENT

- The experimental set up used by Davisson and Germer is as shown in figure to verify wave nature of electrons
- The experiment was performed by varying the voltage from 44 V to 68 V. It was noticed that strong peak appeared in intensity (I) of scattered electron for voltage 54 V at scattering angle 50°





Nickel Crystal

Atoms

Chapter 12

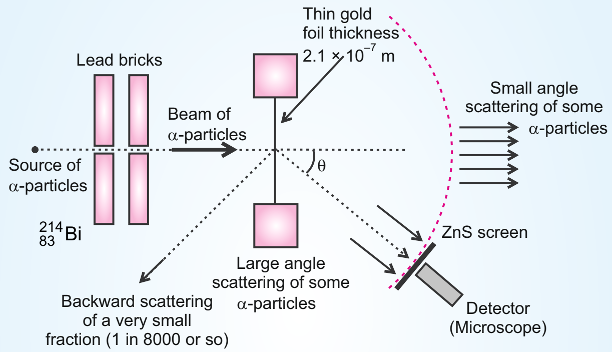
1 ATOMIC MODELS

Thomson's Model | Rutherford Model | Bohr's Model

Thomson's Model

Atom is a spherical cloud of positive charge with electrons embedded into it, like seeds in watermelon.

α-Particle Scattering Experiment and Rutherford nuclear model of atom



Conclusions

1. Only about 0.14% of incident α-particle scatter by more than 1°
2. About 1 in 8000 deflect by more than 90°
3. Size of nucleus to be about 10^{-15} m to 10^{-14} m
4. For large impact parameter the α-particle goes nearly undeviated.
5. In case of head on collision, the impact parameter is minimum and α-particle rebound back ($\theta \simeq \pi$)

Rutherford's Model

According to Rutherford most of the mass of atom and all its positive charge are concentrated in a tiny space of the order of 10^{-14} m, called nucleus and electrons revolve around it. Centripetal force is obtained from electrostatic attraction between electron and nucleus.

Draw backs

(i) Stability of atom

(ii) Line spectrum of atoms

2 IMPACT PARAMETER

It is perpendicular distance of initial velocity vector of the α particle from the centre of nucleus.

$$b = \frac{Ze^2 \cot\left(\frac{\theta}{2}\right)}{4\pi\varepsilon_0 E}$$

Electron orbit

- radius (r) $= \dfrac{e^2}{4\pi\varepsilon_0 mv^2}$
- Kinetic energy (K) $= \dfrac{e^2}{8\pi\varepsilon_0 r}$
- Potential energy (U) $= -\dfrac{e^2}{4\pi\varepsilon_0 r}$
- Total energy (E) = $K + U = -\dfrac{e^2}{8\pi\varepsilon_0 r}$

Bohr's Model

Bohr combined classical and quantum concepts and gave the theory in terms of three postulates.

1. An electron can revolve in certain stable orbits without emission of radiant energy.
2. Electron can revolve only in those orbits in which angular momentum is integral multiple of $\left(\dfrac{h}{2\pi}\right)$

   $L = mv_n r_n = \dfrac{nh}{2\pi}, \quad n = 1, 2, 3, \ldots$
3. When an electron makes a transition from one of the specified non radiatory orbit to another lower energy orbit then radiate energy equal to the difference of energy equal to final and initial state.

- Bohr's model is applicable for hydrogen and hydrogen like elements.

Limitations of Bohr's Model

- Bohr's model is applicable for single electron atom/ions.
- Bohr's model correctly predict the frequencies of the light emitted by hydrogenic (hydrogen like) atoms but unable to explain the relative intensities of light

3 DIFFERENT QUANTITIES FOR HYDROGEN LIKE ELEMENTS

- **Radius of the n^{th} orbit:**

$$r_n = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right)\frac{n^2}{Z} = 0.529\frac{n^2}{Z}\text{Å}$$

$$\Rightarrow r_n \propto \frac{n^2}{Z}$$

- **Speed of electron in n^{th} orbit:**

$$v_n = \frac{e^2}{2h\varepsilon_0}\frac{Z}{n} = \frac{C}{137}\frac{Z}{n}$$

$$\Rightarrow v_n \propto \frac{Z}{n}$$

- **Energy of electron in n^{th} orbit**

$$E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{Z^2}{n^2}\text{ J} = (2.18\times10^{-18})\frac{Z^2}{n^2}\text{ J}$$

or $$E_n = -\frac{13.6Z^2}{n^2}\text{ eV}$$

$$\Rightarrow E_n \propto \frac{Z^2}{n^2}$$

- **Time period of revolution of electron in n^{th} orbit.**

$$T = \left(\frac{4\varepsilon_0 h^3}{me^4}\right)\frac{n^3}{Z^2}$$

$$= \frac{n^3}{Z^2}(1.51\times10^{-16}\text{ s})$$

$$\Rightarrow T \propto \frac{n^3}{Z^2}$$

4 HYDROGEN SPECTRUM

1. Lyman series

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right],\ n = 2, 3, 4 \ldots \infty$$

lies in U.V. region

2. Balmer series

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right],\ n = 3, 4, 5 \ldots \infty$$

Mostly lies in visible region

3. Paschen series

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right],\ n = 4, 5, 6 \ldots \infty$$

lies in near infra red region

4. Bracket series

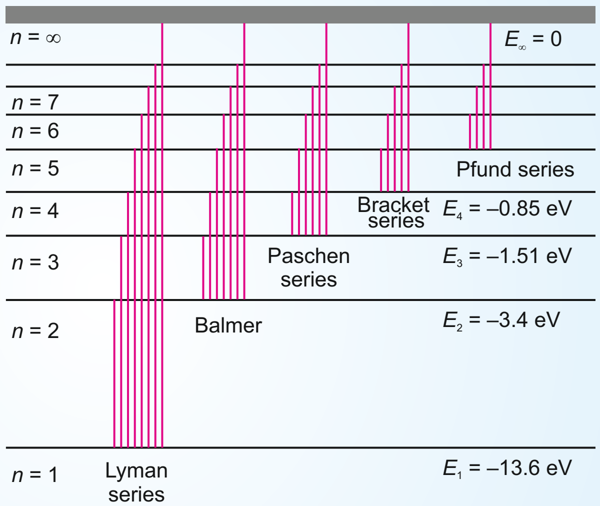
$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n^2}\right],\ n = 5, 6, 7 \ldots \infty$$

lies in infra red region

5. Pfund series

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n^2}\right],\ n = 6, 7, 8 \ldots \infty$$

lies in far infra red region



(Line spectra originate in Transition between energy levels)

Rydberg constant

$$R = \frac{me^4}{8\varepsilon_0 h^3 c} = 1.03 \times 10^7\ \text{m}^{-1} \text{ (By Bohr-model)}$$

$R = 1.097 \times 10^7\ \text{m}^{-1}$ (from Balmer empirical formula)

5 DE BROGLIE'S EXPLANATION OF BOHR'S SECOND POSTULATE OF QUANTISATION

de-Broglie explained second postulate of Bohr's atomic model by assuming an electron has wave nature.

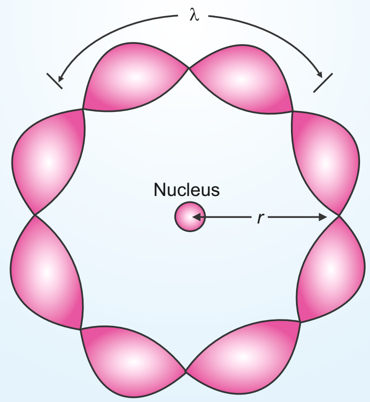
The circumference of orbit should be integer multiple of de-Broglie wavelength of electron in n^{th} orbit.

- $2\pi r_n = n\lambda$, $n = 1,2,3,\ldots$

  or

  $$mv_n r_n = \frac{nh}{2\pi}$$

- This is quantum condition proposed by Bohr for an angular momentum of an electron.



6 LASER

Acronym: light amplification by stimulated emission of radiation.

- It involves population inversion.
- It is highly coherent
- Laser light is highly monochromatic
- Divergence of laser beam is very less
- If there are N atoms, each emitting light with intensity I, then net intensity produced by ordinary source is proportional to NI whereas in laser source, it is proportional to N^2I
- There are low power lasers with a power of 0.5 mW, called pencil lasers which serve as pointers. There lasers are used for delicate surgery of eye or glands in stomach.
- Laser can cut and weld steel.

Nuclei

13 Chapter

1 ATOMIC MASSES AND COMPOSITION OF NUCLEUS

- Before discovery of neutron, nucleus was assumed to be made up of protons and electrons but later this was ruled out using argument of quantum theory.

2 DISCOVERY OF NEUTRON

- In 1932 James Chadwick observed emission of neutral radiation, when beryllium nuclei was bombarded with α-particle on the basis of energy and momentum conservation. Chadwick concluded that it was a new type of neutral particle called neutron.
  $^{9}_{4}Be + ^{4}_{2}He \rightarrow ^{12}_{6}C + ^{1}_{0}n$
- All nuclides with same atomic number but having different mass are called isotopes.
- All nuclides with same mass number are called isobars.
- Nuclides with same neutron but different atomic number are called isotones.
- $A = Z + N$

3 SIZE OF NUCLEUS

- The radius of nucleus with mass number 'A' is $R = R_0A^{1/3}$ where, $R_0 = 1.2 \times 10^{-15}$ m.
- Density of nucleus is approximately 2.3×10^{17} kg/m^3 and is independent of mass number.

4 MASS ENERGY AND NUCLEAR BINDING ENERGY

- **Mass energy :** Einstein showed that mass is another form of energy and one can convert into other form. Einstein gave the famous mass energy equivalence relation $E = mC^2$.
- $1u = 931.5$ MeV/C^2

7 LAW OF RADIOACTIVE DECAY

- Rate of disintegration, $\frac{dN}{dt} = -\lambda N$
- $N = N_0e^{-\lambda t}$
- Half life : $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{\lambda}$
- λ = Decay constant.
- Mean life, $\tau = \frac{1}{\lambda} = 1.44T_{1/2}$

  α-Decay : During α-decay, atomic number decreases by two and mass number by four.
- It is nuclei of helium
  $^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He + Q$
  $Q = (m_x - m_y - m_{He})\,C^2$
- **β-Decay :**
  (i) β^- decay: $^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + e^- + \bar{\nu}$
  $$Q = \left[m\left(_ZX^A\right) - m\left(_{Z+1}Y^A\right)\right]C^2$$
  (ii) β^+ decay: $^{A}_{Z}X \rightarrow ^{A}_{Z-1}Y + e^+ + \nu$
  $$Q = \left[m\left(_ZX^A\right) - m\left(_{Z-1}Y^A\right) - 2m_e\right]C^2$$

  γ-Decay :
- Like an atom, a nucleus also has discrete energy levels, the ground state and excited states. When a nucleus in an excited state spontaneously decays to ground state (or to lower energy state), a photon is emitted. This is called γ-decay.
- The difference in nuclear energy levels is of the order of MeV.

6 NUCLEAR FORCE

- Inside the nucleus, a large attractive force is required to bind the nucleons against repulsion. The force is called nuclear force.
- It is strongest attractive force.
- It is charge independent force i.e. $F_{P-P} = F_{n-n} = F_{P-n}$
- It is short range force.
- It has property of saturation.
- For a distance ($r < 0.8$ fm) it is repulsive force.

8 NUCLEAR FISSION

- When a slow moving neutron strikes a heavy nucleus, which breaks into two intermediate mass nuclear fragments. This is called nuclear fission.
  $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^{1}_{0}n$
- The energy released (the Q-value) in the fission of single uranium is of the order of 200 MeV.
- Multiplication factor $(K) = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$
- Uncontrolled chain reaction is the principle of atom bomb.
- Controlled chain reaction is the principle of nuclear reactors.

9 NUCLEAR REACTOR

- U^{235} or Pu^{239} is used as fuel in a nuclear reactor.
- D_2O, graphite and beryllium oxide are used as moderator to slow down the fast neutrons.
- Rate of reaction is controlled by control rods made of cadmium or boron
- Air, ice cold water, molten sodium or CO_2 are used as coolant.

5 NUCLEAR BINDING ENERGY

- The difference in mass of a nucleus $(_ZX^A)$ and its constituents, ΔM, is called the mass defect.
  $\Delta M = [Zm_P + (A-Z)m_n] - M$
- If one wants to break the nucleus into protons and neutrons. This extra energy $(\Delta M)c^2$, has to be supplied. This energy called binding energy.
  $E_b = \Delta Mc^2$
- Binding energy per nucleon is the measure of stability of nucleus.
- The binding energy per nucleon is practically constant for nuclei of middle mass number ($30 < A < 170$), with maximum of 8.75 MeV for $A = 56$ and has a value of 7.6 MeV for $A = 238$.
- Binding energy per nucleon is lower for both light nuclei ($A < 30$) and heavy nuclei ($A > 170$)

10 NUCLEAR FUSION

It is the phenomenon in which two or more lighter nuclei combine to form a single middle weight nucleus.

Some examples of nuclear fusion.

$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + e^+ + \nu + 0.42$ MeV

$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + n + 3.27$ MeV

$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{1}H + ^{1}_{1}H + 4.03$ MeV

Semiconductor Electronics : Material, Devices and Simple Circuits

14 Chapter

1 SEMICONDUCTOR

- Its conductivity or resistivity lies between conductor and insulator
- Negative temperature coefficient of resistance
- Band gap is less than 3 eV

Intrinsic Semiconductor
- It is pure semiconductor
- Carriers are thermally generated
- At 0 K, behaves like insulator
- $n_e = n_h = n_i$

Extrinsic Semiconductor
- Conductivity is increased by doping.

p-type semiconductor
- Doped with trivalent atom
- Holes are majority carriers
- $n_h >> n_e$

n-type semiconductor
- Doped with pentavalent atom
- Electrons are majority carriers
- $n_e >> n_h$
- Law of mass action $n_e n_h = n_i^2$

2 p-n JUNCTION

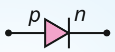
- A p-type semiconductor is brought in contact with a *n*-type semiconductor
- A depletion layer is formed at junction.
- Thickness of depletion layer decreases with increase in doping and forward biasing

Biasing of p-n junction

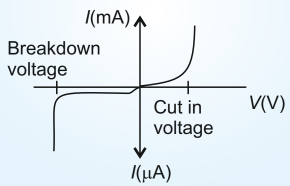
Forward biasing
- Width of depletion layer decreases
- Effective barrier potential decreases
- Low resistance at junction
- Current flown is of the order of mA

Reverse biasing
- Width of depletion layer increases
- Effective barrier potential increases
- High resistance at junction
- Current flown is of the order of μA
- Breakdown occurs at high reverse bias voltage

3 SEMICONDUCTOR DIODE

- It is a device having single p-n junction
- Symbol: Anode  Cathode

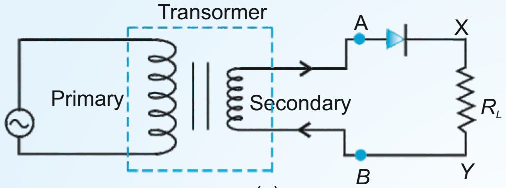
V-I characteristics



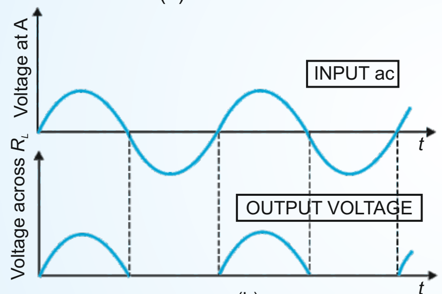
4 APPLICATION OF DIODE

Diode as a Rectifier

Half wave-rectifier
- It rectifies either positive or negative cycle only, of input signal
- Frequency of output and input are same



(a)



(b)

Figure : (a) Half-wave rectifier circuit, (b) Input ac voltage and output voltage waveforms from the rectifier circuit.

Zener diode as a Voltage Regulator

- Zener diode is highly order reverse doped p-n junction diode
- It is operated as regulator when diode is in reversed bias
- The output voltage is fixed and is equal to Zener voltage

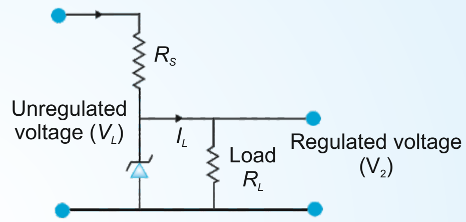
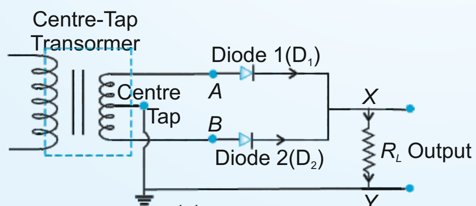


Figure: Zener diode as DC voltage regulator

Full wave rectifier
- It rectifies both the cycles of input
- Frequency of output is two times the frequency of input



(a)

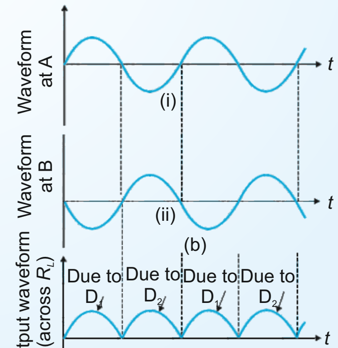


Figure: (a) A Full-wave rectifier circuit; (b) Input wave forms given to the diode D_1 at A and to the diode D_2 at B; (c) Output waveform across the load RL connected in the full-wave rectifier circuit.

5 OPTOELECTRONIC DEVICE

Light Emitting Diode

- It is generally operated in forward bias
- It is used to transmit optical signals
- V-I characteristics is same as of normal p-n junction diode
- E_g should be in range of energy of visible light
- Band width of emitted light is 100 Å to 500 Å

Photodiode

- It is generally operated in reverse bias
- It is used to detect the optical signal
- V-I characteristics lies in 3rd quadrant
- Reverse current increases with increase in intensity of incoming signal

Solar Cell

- It is used in unbiased condition
- It generates emf from solar radiations
- V-I characteristics lies in 4^{th} quadrant
- Semiconductor with E_g closed to 1.5 eV are ideal material for solar cell.

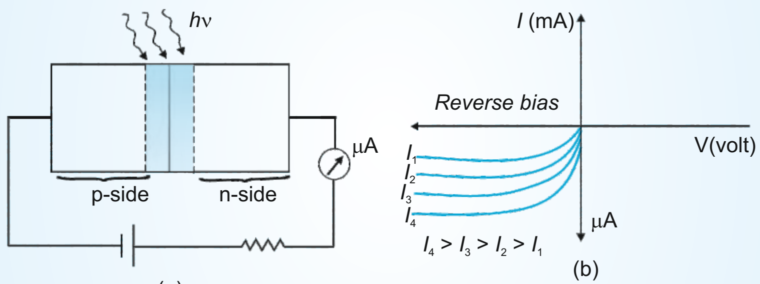


Fig. : (a) An illuminated photodiode under reverse bias, (b) I-V characteristics of a photodiode for different illumination intensity $I_4 > I_3 > I_2 > I_1$

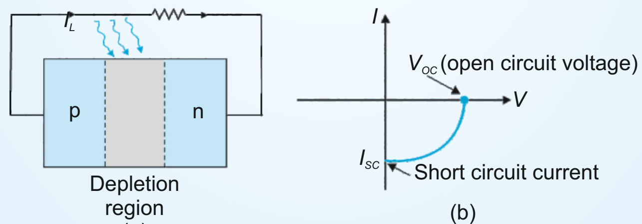


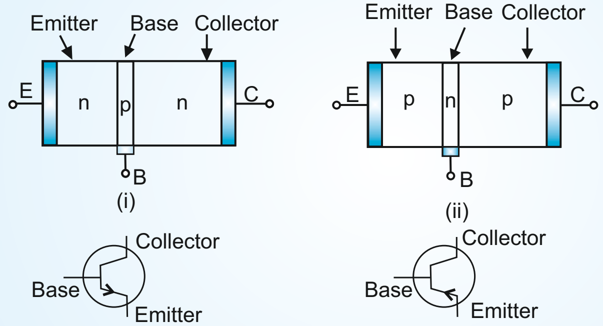
Figure : (a) A typical illuminated p-n junction solar cell; (b) I-V characteristics of a solar cell.

6 JUNCTION TRANSISTOR

- It is two junction and three terminal device
- Fundamental action of transistor is transfer resistor
- Length profile $L_C > L_E > L_B$
- Doping profile $E > C > B$

Types of transistor

- n-p-n transistor
- p-n-p transistor



Modes of Operation

E-B junction	B-C junction	Mode of operation	Application
Forward	Reverse	Active	Amplifier
Forward	Forward	Saturation	Switch (on)
Reverse	Reverse	Cut off	Switch (off)

Configuration of transistor (BJT)

- Common base configuration
- Common emitter configuration
- Common collector configuration

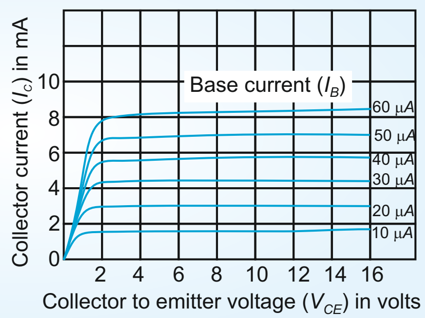
7 COMMON EMITTER (CE) CONFIGURATION

Transistor characteristics

- Input resistance $(r_i)_{CE} = \frac{\Delta V_{BE}}{\Delta I_B}$
- Output resistance $(r_o)_{CE} = \frac{\Delta V_{CE}}{\Delta I_C}$
- Transconductance $(g_m) = \left(\frac{\Delta I_C}{\Delta V_{BE}}\right)$
- Current gain of different configuration

$$\alpha_{ac} = \frac{\Delta I_C}{\Delta I_E},\ \beta_{ac} = \frac{\Delta I_C}{\Delta I_B},\ \gamma_{ac} = \frac{\Delta I_E}{\Delta I_B}$$

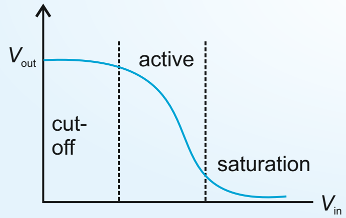
- Output characteristics of CE amplifier



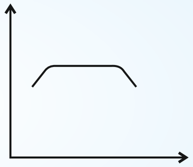
8 APPLICATION OF TRANSISTOR

- **Transistor as an amplifier**
  - Voltage gain $(A_v) = \frac{V_o}{V_i}$

    $= -\beta \frac{R_{out}}{R_{in}} = -g_m R_{out}$
  - Power gain $(A_P) = A_v \times \beta_{ac}$
- Transistor as a switch

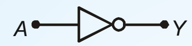
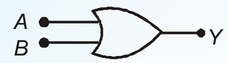
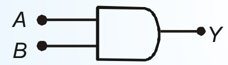
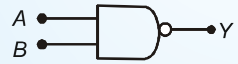
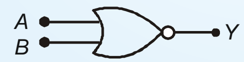


- Variation of voltage gain with frequency is given as



- Transistor as an oscillator
  - Barkhausen criteria for sustained oscillation is $A\beta = 1$

9 LOGIC GATES

Logic gate	Symbol	Characteristic equation	Truth table
NOT		$Y = \overline{A}$	A Y: 0 1; 1 0
OR		$Y = A + B$	A B Y: 0 0 0; 0 1 1; 1 0 1; 1 1 1
AND		$Y = A \bullet B$	A B Y: 0 0 0; 0 1 0; 1 0 0; 1 1 1
NAND		$Y = \overline{A \bullet B}$	A B Y: 0 0 1; 0 1 1; 1 0 1; 1 1 0
NOR		$Y = \overline{A + B}$	A B Y: 0 0 1; 0 1 0; 1 0 0; 1 1 0

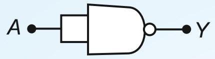
10 BOOLEAN LOGIC

- $A + A = A$
- $A \cdot A = A$
- $A + 1 = 1$
- $A \cdot 1 = A$
- $A + 0 = A$
- $A \cdot 0 = 0$
- $A \cdot \overline{A} = 0$
- $A + \overline{A} = 1$
- $\overline{A + B} = \overline{A} \cdot \overline{B}$
- $\overline{A \cdot B} = \overline{A} + \overline{B}$

11 REALISATION OF BASIC GATES USING NAND OR NOR GATES

Using NAND only

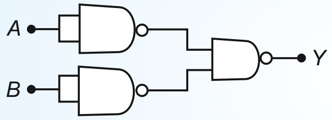
- **NOT**



$Y = \overline{A \cdot A}$

$= \overline{A}$

- **OR**

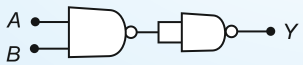


$Y = \overline{\overline{A} \cdot \overline{B}}$

$= \overline{\overline{A + B}}$

$= A + B$

- **AND**

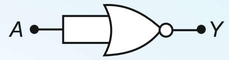


$Y = \overline{\overline{A \cdot B}}$

$= A \cdot B$

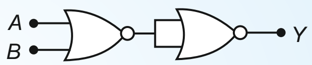
Using NOR only

- **NOT**



$Y = \overline{A \cdot A} = \overline{A}$

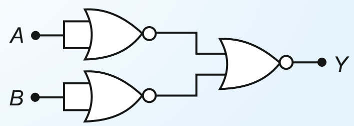
- **OR**



$Y = \overline{\overline{A + B}}$

$= A + B$

- **AND**



$Y = \overline{\overline{A} + \overline{B}}$

$= \overline{\overline{A \cdot B}}$

$= A \cdot B$